NONLINEAR METHOD OF VEHICLE VELOCITY DETERMINATION BASED ON INVERSE SYSTEM AND TENSOR PRODUCT OF LEGENDRE POLYNOMIALS - INTERMEDIATE CLASS

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Abstract

Presented paper discusses two different nonlinear approaches to precrash velocity determination for vehicles from Intermediate Car Class. Data that was used to perform analyses introduced in this paper was taken from National Highway Traffic Safety Administration (NHTSA) database. Such database is comprised of substantial number of crash cases and main focus was put on frontal impacts.

Hitherto used energy methods are based on linear model which proves to be inaccurate and producing significant errors. Presented considerations concern the inverse system and tensor product of Legendre polynomials. The focus of those methods is to establish the value of nonlinear coefficient $b_k$ which is the slope factor of precrash velocity $V_t$ and deformation ratio $C_s$ function.

Keywords: Location, manufacture, protection, VIN, Vehicle Identification Number, vehicle originality

Nomenclature:

$EES$ – Equivalent Energy Speed [m/s]

NHTSA – National Highway Traffic Safety Administration

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1. Introduction

In recent years the development of methods for precrash velocity determination seemed to reach a plateau. Linear models, however simple and showing great ease of use, unfortunately produce significant inaccuracies, reaching up to 30% [39]. Authors focus on developing superior methods in terms of precision, based on nonlinear models, which already have shown promising results in the recent past [18-26].

Presented paper discusses another two nonlinear approaches to precrash velocity determination in Intermediate Car Class. These considerations concern the inverse system and tensor product of Legendre polynomials [1,3]. The focus of those methods is to establish the value of nonlinear coefficient \( b_k \) which is the slope factor. The pre-impact velocity \( V_t \) is a function of said coefficient and deformation ratio \( C_s \) [4,8-12,16-23,20,21,25-28,30,34-37,39]. The \( C_s \) determines the body deformation as an arithmetic mean of deformation depth [23] in specific points \( C_1 \) to \( C_6 \) [13-15,23,26,34,35,37,39].

Once the above parameters are established, the deformation work can be defined. This quantity is then directly used to find the value of EES from the following equation:

\[
EES = \sqrt{\frac{2W}{m}}
\]

where: \( EES \) Equivalent Energy Speed, \( W \) deformation work, \( m \) vehicle mass.

The \( EES \) [6,39,39] defines the amount of energy to deform the vehicle [7,24,38,32,33] while impacting a rigid obstacle, i.e. only plastic deformations occur and the entire kinetic energy accumulated by the vehicle is transferred to deformation work. This is then used as a baseline to find the actual velocity of a vehicle in a real life accident and assess if, for example, collision avoiding maneuvers has been applied [2,23] or if the vehicle speed was properly adjusted according to traffic conditions [34]. Analyzed set of cases focuses on frontal impacts [35,36].

The first method of \( EES \) determination is based on a simple observation, that coefficient \( b_k \) dependent on \( C_s \) behaves similarly to function \( \frac{1}{x} \). This led Authors to consideration of
In order to find the minimum, the function has to be differentiated with respect to Adriena-Marie Legendre were working on the topic.

The second method focuses on Legendre polynomials, which are orthogonal function in the domain of \([-1,1]\). For the purpose of Octave implementation, the set of Legendre polynomials was rescaled and renumbered. Finally, the Legendre polynomials tensor product is considered and the least square method is applied.

### 2. Inverse system method description

Assuming that there are given points \((x_n,y_n)\)\(_{n=1}^{N}\) and a function set \((f_m)_{m=1}^{M}\). The goal is to find coefficients \((a_m)_{m=1}^{M}\), that minimize the value of:

\[
\sum_{n=1}^{N} \left( y_n - \sum_{m=0}^{M} a_m f(x_n) \right)^2 \tag{2}
\]

This problem is called the least-square function approximation and is very well covered in the literature [2]. The first author of this solution was not established, since both Carl Gauss and Adriena-Marie Legendre were working on the topic.

In order to find the minimum, the function has to be differentiated with respect to \(a_k, k = 1, \ldots, M\) obtaining a series of equations

\[
\forall k=1,\ldots,M \sum_{m=1}^{N} a_m \left( \sum_{n=1}^{N} f_k(x_n)f_m(x_n) \right) = \sum_{n=1}^{N} y_n f_k(x_n) \tag{3}
\]

It is convenient to use the matrix notation:

\[
\begin{pmatrix}
\sum_{n=1}^{N} f_1(x_n)f_1(x_n) & \cdots & \sum_{n=1}^{N} f_1(x_n)f_M(x_n) \\
\vdots & \ddots & \vdots \\
\sum_{n=1}^{N} f_M(x_n)f_1(x_n) & \cdots & \sum_{n=1}^{N} f_M(x_n)f_M(x_n)
\end{pmatrix}
\begin{pmatrix}
a_1 \\
\vdots \\
a_M
\end{pmatrix}
=
\begin{pmatrix}
\sum_{n=1}^{N} y_n f_1(x_n) \\
\vdots \\
\sum_{n=1}^{N} y_n f_M(x_n)
\end{pmatrix} \tag{4}
\]

It is worth noticing that matrix \(M\) on LHS (2) is symmetric in sense, that \(M(k,m) = M(m,k)\) for arbitrary \(k, m = 1, \ldots M\). This obvious observation allows to speed up the calculations almost twice. In that stage, the solution of least square problem is down to solving of the \(M\) matrix and making following observations, on condition that it is inverse:

\[
\begin{pmatrix}
a_1 \\
\vdots \\
a_M
\end{pmatrix}
=
\begin{pmatrix}
\sum_{n=1}^{N} f_1(x_n)f_1(x_n) & \cdots & \sum_{n=1}^{N} f_1(x_n)f_M(x_n) \\
\vdots & \ddots & \vdots \\
\sum_{n=1}^{N} f_M(x_n)f_1(x_n) & \cdots & \sum_{n=1}^{N} f_M(x_n)f_M(x_n)
\end{pmatrix}^{-1}
\begin{pmatrix}
\sum_{n=1}^{N} y_n f_1(x_n) \\
\vdots \\
\sum_{n=1}^{N} y_n f_M(x_n)
\end{pmatrix} \tag{5}
\]
Returning to the original problem of finding the $b_k$ as a function of $C_s$, the set of points $(x_n, y_n)_{n=1}^{N}$ is adjacent to $(C_s(n), b_k(n))_{n=1}^{N}$, where $n$ is the number of crash tests. As shown in the preceding sections, the following is true:

\[ f_1(x) = 1, \quad f_2(x) = \frac{1}{x}, \quad f_3(x) = \frac{1}{x^2} \quad \text{and} \quad f_4(x) = \frac{1}{x^3}. \]  

(6)

The solution has the following form:

\[
\begin{pmatrix}
  a_1 \\
  a_2 \\
  a_3 \\
  a_4
\end{pmatrix}
\begin{pmatrix}
  \frac{1}{C_s(n)} \\
  \frac{1}{C_s^2(n)} \\
  \frac{1}{C_s^3(n)} \\
  \frac{1}{C_s^4(n)}
\end{pmatrix}
= \left( \sum_{n=1}^{N} \frac{1}{C_s(n)} \right) \left( \sum_{n=1}^{N} \frac{1}{C_s^2(n)} \right) \left( \sum_{n=1}^{N} \frac{1}{C_s^3(n)} \right) \left( \sum_{n=1}^{N} \frac{1}{C_s^4(n)} \right)^{-1} \left( \sum_{n=1}^{N} b_k(n) \right)
\]

(7)

Whereas the function, that best approximates given points is of following form:

\[ f(C_s) = a_1 + \frac{a_2}{C_s} + \frac{a_3}{C_s^2} + \frac{a_4}{C_s^3} \]

(8)

The rest of the procedure of precrash velocity determination is standard and was already discussed in [22], [24]. As a quick reminder, the subsequent coefficients are defined (with emphasis on crash test number $n$ dependence):

\[ A(n) = m(n) \cdot b_{gs} \cdot \frac{f(C_s(n))}{L_c(n)} \]

(9)

\[ B(n) = m(n) \cdot \frac{f^2(C_s(n))}{L_c(n)} \]

(10)

\[ G(n) = \frac{A^2(n)}{2 \cdot B(n)} \]

(11)

\[ \alpha(n) = C_1(n) + C_2(n) + 2 \cdot (C_2(n) + C_3(n) + C_4(n) + C_5(n)) \]

(12)

\[ \beta(n) = C_1^2(n) + C_2^2(n) + 2 \cdot (C_2^2(n) + C_3^2(n) + C_4^2(n) + C_5^2(n)) + C_1(n) \cdot C_2(n) + C_2(n) \cdot C_3(n) + C_3(n) \cdot C_4(n) + C_4(n) \cdot C_5(n) + C_5(n) \cdot C_6(n) \]

(13)
where \( m \) is the vehicle mass, \( b_{gs} = 3.05 \frac{m}{s} \) is the speed limit and \( L_t \) is the width of dent zone. This yields following relations:

\[
W(n) = \frac{L_t(n)}{5} \left( A(n) \cdot \frac{\alpha(n)}{2} + B \cdot \frac{\beta(n)}{6} + 5 \cdot G \right)
\]

and

\[
EES(n) = \sqrt{\frac{2 \cdot W(n)}{m(n)}}
\]

The relative error is then calculated by following formula:

\[
Error = \left| \frac{V_t(n) - EES(n)}{V_t(n)} \right|
\]

### 3. Inverse system method results

The database used in this considerations consists of 465 crash tests. Authors have used 80% of the database, and then the results are validated. For 80% the following values are given:

\[
\alpha_1 = 0.984709, \alpha_2 = 13.705967, \alpha_3 = -1.218736, \alpha_4 = 0.041163
\]

therefore,

\[
b_k(C_s) = 0.984709 + \frac{13.705967}{C_s} + \frac{-1.218736}{C_s^2} + \frac{0.041163}{C_s^3} \left[ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} \right]
\]

Figure 1 presents the approximation of the inverse method.
For comparison, Figure 2 presents the same data range, but with linear approximation.
The relative error is then calculated by following formula:

(16)

3. Inverse system method results

The database used in this considerations consists of 465 crash tests. Authors have used 80% of the database, and then the results are validated. For 80% the following values are given:

(17)

therefore,

(18)

Figure 1 presents the approximation of the inverse method

Figure 1. Inverse system least square approximation of .

For comparison, Figure 2 presents the same data range, but with linear approximation.

Figure 2. Linear least square approximation of .

There is no doubt that the nonlinear model is the superior one. Figures 3 and 4 present the relative error for nonlinear and linear method respectively.

Figure 3. Relative error of nonlinear model (inverse system).

Figure 4. Relative error of linear model.
The average error of the inverse method equals to 6.3355% whereas, the linear model produces average error of 7.2675%.

Finally, a plot comparing the nonlinear and linear models is presented in Figure 5.

The detailed numerical data is enclosed in the Table 1.

Table 1. Detailed numerical values for inverse system method.

<table>
<thead>
<tr>
<th>$m$</th>
<th>$L_t$</th>
<th>$C_s$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>Linear error</th>
<th>Nonlinear error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1737</td>
<td>1943</td>
<td>0.076665</td>
<td>0.766650</td>
<td>0.088234</td>
<td>0.196983</td>
<td>0.030512</td>
</tr>
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<td>1685</td>
<td>1786</td>
<td>0.530300</td>
<td>5.303000</td>
<td>4.224565</td>
<td>0.008652</td>
<td>0.027690</td>
</tr>
<tr>
<td>1485</td>
<td>1549</td>
<td>0.638900</td>
<td>6.389000</td>
<td>6.129849</td>
<td>0.066540</td>
<td>0.002980</td>
</tr>
<tr>
<td>1619</td>
<td>1486</td>
<td>0.688300</td>
<td>6.883000</td>
<td>7.130574</td>
<td>0.121735</td>
<td>0.010160</td>
</tr>
<tr>
<td>1529</td>
<td>1351</td>
<td>0.635800</td>
<td>6.358000</td>
<td>6.076609</td>
<td>0.063555</td>
<td>0.002527</td>
</tr>
<tr>
<td>1542</td>
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<td>0.690900</td>
<td>6.909000</td>
<td>7.169166</td>
<td>0.112872</td>
<td>0.024523</td>
</tr>
<tr>
<td>1693</td>
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<td>0.656100</td>
<td>6.561000</td>
<td>6.461549</td>
<td>0.087242</td>
<td>0.193653</td>
</tr>
<tr>
<td>1524</td>
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<td>0.608900</td>
<td>6.089000</td>
<td>5.579293</td>
<td>0.042444</td>
<td>0.004111</td>
</tr>
<tr>
<td>1760</td>
<td>1595</td>
<td>0.427300</td>
<td>4.273000</td>
<td>2.749615</td>
<td>0.166005</td>
<td>0.114473</td>
</tr>
<tr>
<td>1650</td>
<td>1539</td>
<td>0.510300</td>
<td>5.103000</td>
<td>3.910029</td>
<td>0.175258</td>
<td>0.142013</td>
</tr>
<tr>
<td>1510</td>
<td>1725</td>
<td>0.374340</td>
<td>3.743400</td>
<td>2.424926</td>
<td>0.012968</td>
<td>0.050547</td>
</tr>
</tbody>
</table>
Let us assume that there are given points \((x_n, y_n, z_n)\)\(^N\)\(_{n=1}^N\) and function family \((h_m)_{m=1}^M\) (functions of two variables). Again, the objective is to obtain the coefficients \((a_m)_{m=1}^M\), which minimize its value.

\[
\sum_{n=1}^{N} \left( z_n - \sum_{m=1}^{M} a_m h(x_n, y_n) \right)^2
\]  

(20)

Similarly to section 2, the issue of least square approximation is reduced to a linear solution:

\[
\begin{pmatrix}
\sum_{n=1}^{N} h_1(x_n, y_n) \cdots \sum_{n=1}^{N} h_M(x_n, y_n) \\
\vdots \\
\sum_{n=1}^{N} h_1(x_n, y_n) \cdot h_1(x_n, y_n) & \cdots & \sum_{n=1}^{N} h_M(x_n, y_n) \cdot h_M(x_n, y_n)
\end{pmatrix}
\begin{pmatrix}
a_1 \\
\vdots \\
a_M
\end{pmatrix}
= 
\begin{pmatrix}
\sum_{n=1}^{N} y_n h_1(x_n) \\
\vdots \\
\sum_{n=1}^{N} y_n h_M(x_n)
\end{pmatrix}
\]  

(21)
As for the choice of function \((h_m)_{n=1}^N\), Legendre polynomial product tensors are chosen. Consideration involves a sequence of polynomials \((P_m)\) defined by a iterative formula:

\[
\forall_{m \geq 1} \, (m+1)P_{m+1}(x) = (2m+1)x \cdot P_m(x) - nP_{m-1}(x)
\]  

(22)

where \(P_0(x) = 1\) and \(P_1(x) = x\) These are Legendre polynomials from a range of \([-1,1]\). First Legendre polynomials are:

\[
P_0(x) = 1, \quad P_1(x) = \frac{1}{2} (3x^2 - 1), \quad P_2(x) = \frac{1}{2} (5x^3 - 3x), 
\]  

(23)

Legendre polynomials have a feature called orthogonality:

\[
\forall_{i \neq j} \int_{-1}^{1} P_i(x) P_j(x) \, dx = 0
\]  

(24)

This feature is a natural consequence of Legendre polynomials being created as a result of orthogonalization of Gram-Schmidt function family \(\{1,x,x^2,x^3, \ldots\}\). Orthogonality is valuable in this case, because the matrix \(M\) on left hand side (4) is closer to diagonal matrix. This results in smaller error of coefficients \((a_m)_{n=1}^N\).

In this application, Legendre polynomials sequence is renumbered so \(Q_m = P_{m-1}\). Then following is obtained:

\[
\forall_{m \geq 3} \, (m-1)Q_m(x) = (2m-3)x \cdot Q_{m-1}(x) - (m-2)Q_{m-2}(x)
\]  

(25)

where \(Q_1(x) = 1\) and \(Q_2(x) = x\). To rescale the polynomials for arbitrary interval \([a,b]\), the following relation is used:

\[
f_m(x) = Q_m \left(\frac{2x-a-b}{b-a}\right)
\]  

(26)

Finally, the tensor product of two function \(f, g\) is described as:

\[
h(x,y) = f \otimes g(x,y) = f(x)g(y)
\]  

(27)

In this application the \((f_j)_{j=1}^5\) and \((g_j)_{j=1}^5\) are the first five Legendre polynomials. This gives 25 tensor products.

\[
\begin{align*}
h_1 &= f_1 \otimes g_1, \quad h_2 = f_1 \otimes g_2, \quad h_3 = f_1 \otimes g_3, \quad h_4 = f_1 \otimes g_4, \quad h_5 = f_1 \otimes g_5, \\
h_6 &= f_2 \otimes g_1, \quad h_7 = f_2 \otimes g_2, \quad h_8 = f_2 \otimes g_3, \quad h_9 = f_2 \otimes g_4, \quad h_{10} = f_2 \otimes g_5, \\
h_{11} &= f_3 \otimes g_1, \quad h_{12} = f_3 \otimes g_2, \quad h_{13} = f_3 \otimes g_3, \quad h_{14} = f_3 \otimes g_4, \quad h_{15} = f_3 \otimes g_5, \\
h_{16} &= f_4 \otimes g_1, \quad h_{17} = f_4 \otimes g_2, \quad h_{18} = f_4 \otimes g_3, \quad h_{19} = f_4 \otimes g_4, \quad h_{20} = f_4 \otimes g_5, \\
h_{21} &= f_5 \otimes g_1, \quad h_{22} = f_5 \otimes g_2, \quad h_{23} = f_5 \otimes g_3, \quad h_{24} = f_5 \otimes g_4, \quad h_{25} = f_5 \otimes g_5.
\end{align*}
\]
5. Result of tensor product method

The database consists of 465 crash tests. Model is created upon 80% of records and then validated. The program returns following values:

\[
\begin{align*}
\alpha_1 &= 13.838896, \alpha_2 = 2.748867, \alpha_3 = -1.584037, \alpha_4 = -0.167105, \alpha_5 = 0.311362, \\
\alpha_6 &= -0.120319, \alpha_7 = 0.277501, \alpha_8 = -0.282229, \alpha_9 = -0.110090, \alpha_{10} = 0.555082, \\
\alpha_{11} &= -0.187698, \alpha_{12} = 0.248710, \alpha_{13} = -0.038370, \alpha_{14} = -0.025183, \alpha_{15} = 0.161465, \\
\alpha_{16} &= -0.774463, \alpha_{17} = 1.230121, \alpha_{18} = -2.217478, \alpha_{19} = 0.766343, \alpha_{20} = -0.364707, \\
\alpha_{21} &= 0.442425, \alpha_{22} = -1.410082, \alpha_{23} = 2.156430, \alpha_{24} = -1.706919, \alpha_{25} = 1.989650.
\end{align*}
\]

The plot of Legendre polynomials tensor product approximation is presented in Figure 6.

![Performance of linear and nonlinear models](image)

**Figure 6. Tensor product approximation by orthogonal Legendre polynomials.**

It is clearly visible that it has big advantage over the linear model presented in Figure 7.
The average value of relative error is $0.058355 \approx 5.8355\%$ as presented in Figure 8. Compared to result from section 3, this still shows an improvement in accuracy. Moreover, it is a better result than the linear model, where the relative error was $0.069791 \approx 6.9791\%$ as shown in Figure 9.
Finally, a comparison of linear and Legendre approach is presented in Figure 10.
Table 2 presents detailed data of Legendre approach.

**Table 2. Detailed numerical values of the inverse method.**

<table>
<thead>
<tr>
<th>$m$</th>
<th>$C_s$</th>
<th>$V_t$</th>
<th>Expected linear</th>
<th>Expected nonlinear</th>
<th>Linear error</th>
<th>Nonlinear error</th>
</tr>
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<td>11.758387</td>
<td>8.020497</td>
<td>0.434922</td>
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6. Conclusions

As the preceding sections clearly demonstrate, the nonlinear approach shows promising results. The advantage is best visible in the figures 6 and 7, where Authors put emphasis on the data structure itself. That is the crucial aspect when analyzing data and shows evident advantage of the nonlinear over the linear approach.

When comparing these methods to the other papers done by the Authors, it does not show such an extensive improvement. This may indicate that this particular approach might need some further development in order to achieve comparable or even lower error values. Nevertheless, the improvement is clearly visible, especially when considering the whole spectrum of examined cases. This is due to the fact of applying spline functions, which estimate the EES speed with larger error, although the error is distributed more evenly throughout the analyzed vehicles.

7. References


