

TO DETERMINE THE STABILITY OF THE METROBUS IN UNSTABLE DRIVING MODES

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Abstract

Recently in many cities of the world began to introduce so-called «metrobus» or BRT (Bus Rapid Transit) systems, which became a cheaper alternative to the metro and other rail transport, in particular trams. The aim of the work is to determine the stability indicators of the metrobus in the transitional traffic modes, in particular when performing such manoeuvres as «steering wheel jerk» and «shuffle». For this purpose, the equations of metrobus plane-parallel motion are supplemented by equations of the links of the road train in the vertical plane by halopilation (tangage, trim) and roll. The critical straight-line speed of the three-link metrobus has been determined, which was 32,06 m/s, and this speed is independent of the corners of the steering wheels of the bus and the trailer links. It has been shown that as the steering wheel angle of the bus increases, the difference in the folding angles of the bus increases, with the second folding angle significantly exceeding the first, especially considering the roll of the metrobus body. It has been established that the greatest rolls and loads of the metrobus axles are those of the last trailer, which is the limiting factor for the critical speed. The lateral accelerations of individual metrobus links and their yaw velocity when performing the manoeuvre «steering wheel jerk» and «shuffle, $S_1 = 24 \text{ m}$ », show that both the bus and the second trailer link are a limiting factor when performing various manoeuvres, but the acceleration value does not exceed the permissible 0.4 g, so its stability under these conditions is ensured.

Keywords: metrobus roll; acceleration; stability; trailer

1. Introduction

The last time many cities in the world began to introduce so-called buses or BRT systems, that have become a cheaper alternative to the subway and other rail transport,

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in particular trams. BRT transport is already operating in more than 200 cities around the world.

Therefore, we consider it appropriate to study this experience and the advantages of the urban transport system, possibility of implementation in Ukraine. After all, the appearance of the metrobuses will help to evolutionarily displace the minibus from Ukrainian cities and switch to a more progressive model of urban transport functioning.

Metrobus line is usually plying buses of a particularly large capacity and length (18 m, 22 m, 24 m or 25 m). Their main difference from the usual city routes is that the metrobuses run in a separate dedicated lane with short intervals. Also an important component of the BRT system are stops, which are special terminals (stations) equipped with turnstiles where fares are paid [8].

Another advantage of BRT systems is the speed of construction of such lines, which can be used by existing highways in cities. Usually such a line is built from (1 to 2) years, while the construction of a subway, tram lines can last from (3 to 10) years.

The metrobuses received a special development with the advent of three-link buses, Figure 1, that can carry up to 300 passengers at a time, compared to 180 passengers in two-link transports. Thus, having three-link buses that move at short intervals of (1-5) minutes, the metrobus line can solve the transport problems of many Ukrainian cities [8].



Fig. 1. Three-link metrobus

Safe movement of any vehicle, including the metrobus is largely determined by its dynamic properties, and to a large extent its stability and road handling [12]. At present time, the problem of determining the stability of road trains has been well studied. Thus, a simplified analysis of the manoeuvrability and stability of combinations of vehicles such as a tractor unit combined with one or two semi-trailers or a truck and a full trailer has been carried out in work [5, 10]. Car combinations are considered as linear dynamic systems with two degrees of freedom for each block. The equations of motion

are derived from the effect of braking and acceleration and a special equation is obtained for the constant speed movement. In work [7] three-dimensional dynamic models of the car and the trailer have been developed, on the basis of which a dynamic model of the train has been built. Based on the approximation theory of the first order of ordinary differential equations and Hopf's bifurcation theory, the linear and non-linear stability of each element and of the automobile train as a whole in straight motion is studied. The numerical results show that for non-linear and linear models, critical speeds differ little. In work [2] the equation of vertical and lateral dynamics of a road vehicle with 6 degrees of freedom is reduced to a matrix form. The movement of such a means in the vertical and lateral planes is investigated. It is shown that the developed method can be applied to the analysis of the stability of traffic, in particular passenger trains. The work [1] considered the multivariate extension of the D2-IBC method (Data Driven - Inversion Based Control) and discussed in detail its application with regard to control of the stability of road trains. In work [4] the model of the train with 31 degrees of freedom with the help of the AutoSim package, and directions of improvement of the train stability are shown. It is shown that its stability can be significantly improved by means of an inert, which is considered effective for increasing stability and productivity of multi-link trains. However, as experience has shown, the determination of the nature of the behaviour of the system in the area of volatility and the identification of its causes is still relevant.

The manoeuvrability and stability characteristics of a motor vehicle are known to be determined by a combination of the operational, mass-geometric and structural parameters of its modules (for metrobus it is bus and trailers) and their control systems. In general, the desired messages of the above parameters in terms of stability, even for the same vehicle, vary over the range of operating loads and speeds. As a result, it is difficult to obtain, in the early stages of the development of motor vehicle, precise design parameters and quantitative indicators for the sustainability of its movement. Success in solving such problems depends on the success of the mathematical model and its essential parameters describing the behavior of the dynamic system in different modes of motion.

The work [9] has developed differential equations of plane-parallel motion to determine manoeuvrability and stability, but these equations can only characterize the stability of a motor vehicle in straight motion. Their use to assess the stability of a motor vehicle in the transitional traffic modes may lead to significant errors. The purpose of the work is therefore to determine the stability indicators of the metrobus in the transitional traffic modes, in particular when performing such manoeuvres as «steering wheel jerk» and «shuffle».

2. Materials and methods

When researching the stability of road train movement, they are usually considered plane-parallel motion of links. At the same time, it is believed that the normal reactions of the support surface to the wheels of the starboard side are the same. Under this condition, traffic stability is considered for a flat road train model.

The paper [9] obtained a system of differential equations, which describes the plane-parallel motion of the links of three-link swing-jointed bus, Figure1. This system of equations is written as:

- by variable “v”

$$(m_1 + m_2 + m)(\dot{v} - u\omega) - [(m_1 d_1 + m_2 l_1) \sin \varphi_1 + m_2 d_2 \sin(\varphi_1 + \varphi_2)] \times [(\dot{\omega} - \dot{\varphi}_1) + m_2 d_2 \sin(\varphi_1 + \varphi_2)] \dot{\varphi}_2 + c(m_1 + m_2)\omega^2 + (m_1 d_1 + m_2 l_1)(\omega - \dot{\varphi}_1)^2 \cos \varphi_1 + m_2 d_2(\omega - \dot{\varphi}_1 - \dot{\varphi}_2)^2 \cos(\varphi_1 + \varphi_2) = -(X_1 \cos \theta_1 + Y_1 \sin \theta_1) - X_2 - X_3 \cos(\theta_2 + \varphi_1) - Y_3 \sin(\theta_2 + \varphi_1) - [X_4 \cos(\theta_3 + \varphi_2 + \varphi_1) - Y_4 \sin(\theta_3 + \varphi_2 + \varphi_1)];$$

- by variable “u”

$$(m_1 + m_2 + m)(\dot{u} - v\omega) - (m_1 + m_2)c\dot{\omega} - [(m_1 d_1 + m_2 l_1)m_2 d_2 \cos \varphi_1 + m_2 d_2 \cos(\varphi_1 + \varphi_2)] \times [(\dot{\omega} - \dot{\varphi}_1) + m_2 d_2 \cos(\varphi_1 + \varphi_2)] \dot{\varphi}_2 - (m_1 d_1 + m_2 l_1)(\omega - \dot{\varphi}_1)^2 \sin \varphi_1 - m_2 d_2(\omega - \dot{\varphi}_1 - \dot{\varphi}_2)^2 \sin(\varphi_1 + \varphi_2) = -(X_1 \sin \theta_1 - Y_1 \cos \theta_1) + Y_2 + X_3 \sin(\theta_2 + \varphi_1) + Y_3 \cos(\theta_2 + \varphi_1) + [X_4 \sin(\theta_3 + \varphi_2 + \varphi_1) - Y_4 \cos(\theta_3 + \varphi_2 + \varphi_1)];$$

- by variable “ω”

$$[I + (m_1 + m_2)c^2 + (m_1 d_1 + m_2 l_1) \times c \times \cos \varphi_1 + m_2 \times d_2 \times c \times \cos(\varphi_1 + \varphi_2)] \dot{\omega} - (m_1 + m_2) \times c \times (\dot{u} + \omega v) = -M_{C1} - a(X_1 \sin \theta_1 - Y_1 \cos \theta_1) - bY_2 - c[X_3 \sin(\theta_2 + \varphi_1) + Y_3 \cos(\theta_2 + \varphi_1)] - m_2 d_2(\omega - \dot{\varphi}_1 - \dot{\varphi}_2)^2 \sin(\varphi_1 + \varphi_2) - c[X_4 \sin(\theta_3 + \varphi_1 + \varphi_2) + Y_4 \cos(\theta_3 + \varphi_1 + \varphi_2)];$$

- by variable “φ₁”

$$(I_1 + m_1 d_2 + m_2 l_1^2 + m_2 d_2 l_1 \cos \varphi_2) \times (\dot{\varphi}_1 - \dot{\omega}) + (m_1 d_1 + m_2 l_1) \times [\dot{v} \sin \varphi_1 + (\dot{u} - \dot{\omega} c) \cos \varphi_1] + m_2 d_2 l_1 \cos \varphi_2 \dot{\varphi}_2 + (m_1 d_1 + m_2 l_1) \times \omega \times [v \cos \varphi_1 - (u - c\omega) \sin \varphi_1] - m_2 d_2 l_1(\omega - \dot{\varphi}_1 - \dot{\varphi}_2)^2 \sin \varphi_2 = -M_{C2} + (d_1 + b_{1\rho}) \times (X_{3\rho} \sin \theta_{1\rho} + Y_{3\rho} \cos \theta_{1\rho}) + l_1 \times [X_4 \sin(\theta_3 + \varphi_2) + Y_4 \cos(\theta_3 + \varphi_2)];$$

- by variable “φ₂”

$$(I_2 + m_2 d_2^2) \times (\dot{\varphi}_2 - \dot{\omega}) + m_2 d_2 \times [\dot{v} \sin(\varphi_1 + \varphi_2) + (\dot{u} - \dot{\omega} c) \cos(\varphi_1 + \varphi_2)] + [I_2 + m_2 d_2 \times (d_2 + l_1 \cos \varphi_2)] \dot{\varphi}_1 + (m_2 d_2 l_1 \cos \varphi_2) \dot{\varphi}_1 + m_2 d_2 l_1(\omega - \dot{\varphi}_1)^2 \sin \varphi_2 - m_2 d_2 \omega \times [(u - \omega c) \sin(\varphi_1 + \varphi_2) - v \cos(\varphi_1 + \varphi_2)] = -M_{C3} + (d_2 + b_2) \times (X_4 \sin \theta_3 + Y_4 \cos \theta_3). \quad (1)$$

In the system of equations (1), the following designation are adopted:

M, C, I – mass, centre of mass and centre of inertia of the driving link relative to the vertical axis passing through the centre of masses, p. C;

x, y – absciss and ordinate, p. C in an inertial coordinate system;

m_k, C_k, I_k ($k = 1$ to 2) – same for the first and second driven links;

$\theta, \theta_1, \theta_2$ – heading angles of road train links;

$v = \dot{x}\cos\theta + \dot{y}\sin\theta$, $u = -\dot{x}\sin\theta + \dot{y}\cos\theta$ – speed projection p. C on the longitudinal and transverse axis of the driving link;

M_{c1} , M_{c2} , M_{c3} – moments of resistance to the turning of road train links;

ω , ω_1 , ω_2 – the angular speeds of road train links;

θ_{1i} ($i = 1, \dots, n_1$), $\theta_{1\rho}$ ($\rho = 1, \dots, n_3$), θ_{2s} ($s = 1, \dots, n_4$) – corners of the axes of road train links;

$X_{\alpha\beta}$, $Y_{\alpha\beta}$ – longitudinal and transverse reactions on road train axle wheels;

$a_i = CA_i$, $b_j = CB_j$, $l = A_1B_{n2}$, $c = O_1C$, $d = O_1C_1$, $b_{1\rho} = CB_{1\rho}$, $L_1 = O_1B_{1n3} = d_1 + b_{1n3}$,

$c_1 = O_2C_1$, $l_1 = O_2O_1 = c_1 + d_1$, $b_{2s} = C_2B_{2s}$, $L_2 = O_2B_{2n4} = d_2 + b_{2n4}$ – geometric parameters of the road train.

The resulting system of five differential equations can be used to find five unknown – v , u , ω , φ_1 i φ_2 . The system of equations (1) consists of the longitudinal and lateral reactions of the road to the wheels of individual links of the road train. The magnitude and direction of the X_i longitudinal reactions depend on the movement of the motorway train (acceleration, steady free movement, braking). Lateral reactions, Y_i , depend on the accepted lateral wheel deformation model.

There are various phenomenological theories of spring deformation of wheels, among which the axiomatics of Rokar I. [3] are the most common. If the curve $Y(\delta)$ is convex, Figure 2, the following empirical characteristics will be possible analytical approximations of the sensitivity of the side reaction to the angle of slip.

$$Y = k_0 \arctg(c\delta), Y = k_0 \operatorname{tg}(c\delta), Y = \frac{k\delta}{\sqrt{1+\chi^2\delta^2}} \quad (2)$$

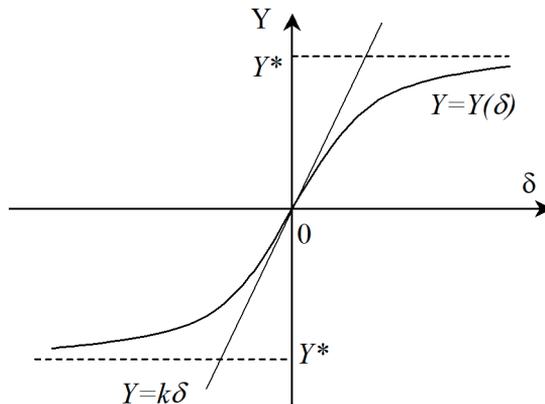


Fig. 2. Lateral force dependence on sideslip angle according to I. Rokar

The general requirement for all curves shown in Figure 3 is that the function is the sum of the alternating series.

$$Y = k\delta - k'\delta^3 + k''\delta^5 \quad (3)$$

Neglecting the redistribution of normal reactions between the wheels of the same axle, we shall replace the wheels of each axle with one corrected wheel centered in the middle of the axle. Then k , k' , k'' , ... will be the given characteristics of the axes. Non-linearity has to be taken into account because if the relationship between the forces acting on one wheel and the angles of its slip can still be considered linear in some range, the relationship between the forces operating on the axle and sideslip angles, in most cases is non-linear, even at low operating forces. This is due to the influence on the sideslip angles of the axes of suspension kinematics, steering characteristics, redistribution of normal reactions.

Take the last of the approximations (2). Since $\lim_{\delta \rightarrow +\infty} Y(\delta) = \frac{k}{\chi} = Y^*$, then $\chi = \frac{k}{Y^*}$

There's $Y^* = \psi \times Z$, where ψ – is the coefficient of transverse adhesion between the tyre and the supporting surface; Z – is the vertical wheel load. Thus, for the lateral reaction we get:

$$Y_i = \frac{k_i \delta_i}{\sqrt{1 + \chi_i^2 \delta_i^2}}, \chi_i = \frac{k_i}{\psi Z_i} \quad (i = 1, 2, 3, 4) \quad (4)$$

Manoeuvring in the traffic limits and moving the metrobus in the designated lanes at a high speed can lead to a significant change in the reactions of the support surface on the wheel of the links of the road train. Therefore, in addition to the horizontal movement of the road train, as described by the differential equations of plane-parallel motion, it is necessary to consider the movement of the road train in the longitudinal vertical and transverse planes. The communication between the undercarriage and the unbreakable masses of the real structure of the road train is carried out by means of flexible and damping devices, and between the unbreakable masses and the road - by means of tyres which are characterized by both resilient and damping properties. At relatively low speeds of the road train under manoeuvring conditions, it can be assumed that the movement of lean and unsprung masses is synchronized. The suspension elements and the tyres shall be supposedly subjected to static compression with little resistance of shock absorbers [11]. Under such circumstances, it can be assumed that the spring-loaded masses oscillate on the elastic elements with the given rigidity.

The calculation diagram, equivalent to the three-link metrobus in the longitudinal vertical plane, can then be presented in the form of a Figure 3, using common symbols.

In this case, the interaction forces in the coupling and traction couplings do not affect the redistribution of loads on the sides of the road train links. Therefore, a fairly complex system-three-links metrobus can be considered as three systems – a bus, the first trailer unit (semi-trailer) and the second trailer unit (trailer) which is rolling independently. The heel axis of each link is also considered to be parallel to the supporting surface, and the movement of the metrobus links in the vertical plane along the angles

of halopilation (tangage, trim) and heel influence lateral motion, primarily and mainly by changing the vertical loads on the wheels, thereby changing the vertical reactions of the supporting surface. In accordance with this concept, a distinction has been drawn between lateral, longitudinal and transverse motion [6].

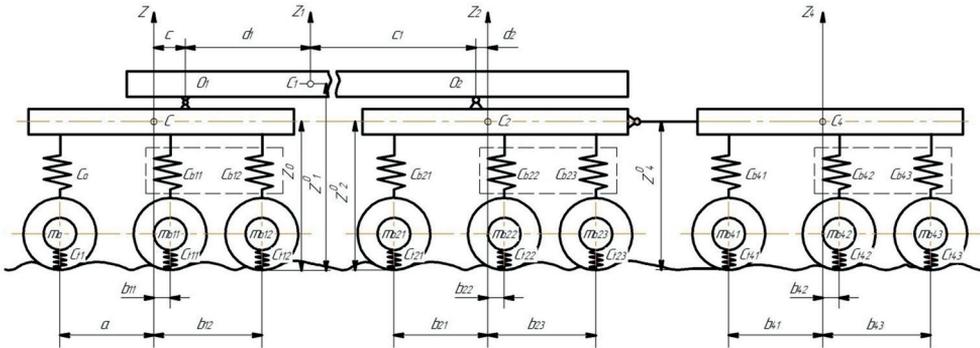


Fig. 3. Diagram of the three-link metrobus

In the work [6] a system of equations is obtained describing the loading and unloading of the sides of the road train when it performs various manoeuvres. This system with some correction can be applied to the metrobus.

The dynamic constituents of vertical reactions in supports caused by roll angles γ and γ_0 , γ_2 and $\gamma_0^{(2)}$, γ_3 and $\gamma_0^{(3)}$ (loading and unloading) shall be defined as [11]:

- for the left side of the metrobus

$$\Delta G_1 = q(\gamma - \gamma_0) \frac{H}{2}; \Delta G_{1i} = q_{1i}(\gamma - \gamma_0) \frac{H_1}{2}; \Delta G_{2j} = q_{2j}(\gamma_2 - \gamma_0^2) \frac{H_2}{2};$$

$$\Delta G_{3\rho} = q_{3\rho}(\gamma_3 - \gamma_0^3) \frac{H_3}{2}; \Delta G_{4s} = q_{4s}(\gamma_3 - \gamma_0^3) \frac{H_3}{2}. \tag{5}$$

- for the right side of the metrobus

$$\Delta G_1^1 = q_A^1(\gamma - \gamma_0) \frac{H}{2}; \Delta G_{1i}^1 = q_{1i}^1(\gamma - \gamma_0) \frac{H_1}{2}; \Delta G_{2j}^1 = q_{2j}^1(\gamma_2 - \gamma_0^2) \frac{H_2}{2};$$

$$\Delta G_{3\rho}^1 = q_{3\rho}^1(\gamma_3 - \gamma_0^3) \frac{H_3}{2}; \Delta G_{4s}^1 = q_{4s}^1(\gamma_3 - \gamma_0^3) \frac{H_3}{2}. \tag{6}$$

where

$$\gamma_0 = \frac{-P_n(0,5H + \varepsilon) + H[(l - z_0) \times (q_A + q_{B1}) - q_A m_{A2} g / C_{t1} - q_{B1} m_{B1} g / (C_{t11} + C_{t12})]}{0,5H^2(q_A + q_{B1})} = \{l - z_0 - [0,5P_n + P_n \varepsilon / H + q_A m_{A2} g / C_{t1} + q_{B1} m_{B1} g / (C_{t11} + C_{t12})] / (q_A + q_{B1})\} \times \frac{2}{H};$$

$$\gamma_0^{(2)} = \frac{-P_{n2}(0,5H_2 + \varepsilon_2) + H_2[(l_2 - z_{02}^{(2)}) \times (q_{A2} + q_{B2}) - q_{B21} m_{B21} g / C_{t21} - q_{B21} m_{B21} g / (C_{t22} + C_{t23})]}{0,5H_2^2(q_{A2} + q_{B2})} = \{l_2 - z_0^2 - [0,5 P_{n2} \varepsilon / H_2 + q_{B21} m_{B21} g / C_{t21} + q_{B2} m_{B2} g / (C_{t22} + C_{t23})] / (q_{B21} + q_{B22})\} \times \frac{2}{H_2};$$

$$\gamma_0^{(3)} = \frac{-P_{n3}(0,5H_3 + \varepsilon_3) + H_3[(l_3 - z_0) \times (q_{A4} + q_{B4}) - q_{A4}m_{A3}g/C_{t4} - q_{B4}m_{B3}g/(C_{t41} + C_{t42})]}{0,5H_3^2(q_{A2} + q_{B2})} = \{l - z_3 - [0,5P_{n3}\varepsilon/H_3 + q_{A4}m_{A3}g/C_{t3} + q_{B1}m_{B4}g/(C_{t41} + C_{t42})]/(q_{A4} + q_{B4})\} \times \frac{2}{H_3} \quad (7)$$

and

$$\gamma = \frac{\gamma_0 0,5H^2(q_A + q_{B1}) + F(Z_0 + h)}{-F\varepsilon + 0,5H^2(q_A + q_{B1})};$$

$$\gamma_2 = \frac{\gamma_0^{(2)} 0,5H_2^2(q_{B21} + q_{B2}) + F_2(Z_0^{(2)} + h_2)}{-F_2\varepsilon_2 + 0,5H_2^2(q_{B21} + q_{B2})};$$

$$\gamma_3 = \frac{\gamma_0^{(3)} 0,5H_3^2(q_{A4} + q_{B4}) + F_3(Z_3 + h_3)}{-F_3\varepsilon_3 + 0,5H_3^2(q_{A4} + q_{B4})} \quad (8)$$

Equations (7) and (8) denote:

P_{ni} – is the gravity of the lightweight certain link of the metrobus;

N_i – track of the metrobus certain link;

ε_i – change of the N_i wheel track for the heel of the i -th certain link metrobus body;

l – bus base;

l_2 – is the distance from the point of coupling of the bus with the first trailer to its centre of mass;

l_3 – distance from the point of coupling of the first trailer link to the centre of mass of the second trailer link;

$z_{0,1,2,3}$ – is the bend of suspension of the i -th certain metrobus axis;

$2m_{ai}, 2m_{bj}$ – uncompressed masses i -th front and j -th rear suspension;

$c_{1i}, c_{2j}, c_{1it}, c_{2jt}$ – respectively, radial rigidities of suspension and tyres;

f_{ai}, f_{bj} – static bends of suspension;

$\lambda_{ai}^0, \lambda_{bj}^0$ – vertical deformation of tyres;

F_{ai}^0, F_{bj}^0 – vertical reactions of the i -th front and j -th rear support on the frame i -th metrobus link;

l_{ai}, l_{bj} – the height of the centres of masses of the metrobus links above the support surface, with the suspension and tyres not being formed;

q_A, q_{B1} – averaged rigidities of the two consecutive elastic suspension elements and the tyres, respectively, of the first and second axles of the bus;

$q_{A2}, q_{B2}, q_{A4}, q_{B4}$ – averaged rigidities of the two successively connected flexible suspension elements and the axle tyres, respectively of the first and second metrobus link;

F, F_2, F_3 – summary vertical reactions of the wheels of one side of the metrobus.

This system may also be used for the metrobus under consideration, provided that the suspension and tyres c_{12} , c_{t12} , c_{21} and c_{23} , c_{t21} and c_{t23} , c_{31} and c_{33} , c_{t31} and c_{t33} are taken to be zero.

Thus, dynamic metrobus loads, taking into account on-board reallocation, will take the form:

$$G_1 = G_1^0 - \Delta G_1; G_2 = G_2^0 - \Delta G_2; G_3 = G_3^0 - \Delta G_3; G_4 = G_4^0 - \Delta G_4;$$

$$G_1^1 = G_1^{01} - \Delta G_1^1; G_2^1 = G_2^{01} - \Delta G_2^1; G_3^1 = G_3^{01} - \Delta G_3^1; G_4^1 = G_4^{01} - \Delta G_4^1 \quad (9)$$

Equation (9) is the basis for calculating the values of the loaded and unloaded wheels of the trailers of the metrobus and for subsequent analysis of the metrobus stability.

In determining the stability of buses, including articulated buses, it is considered that the bus is fully loaded, the mobility of the passengers is absent, and the whole spring mass is a solid body [10].

The basic data for calculating the stability of an articulated bus are: the total mass of the road train – 38,000 kg, the load on the coupling device of the bus and the first trailer – 1,990 kg, the total rigidity of the front wheels of the bus – 640 kN/m, the rear wheels – 950 kN/m, the wheels of the first and second trailers – 840 kN/m, rigidity of the bus tyre – 1250 kN/m; rigidity of the trailer tyre – 980 kN/m; geometric parameters of the road train: length – 26,000 m, width – 2,460 m, height – 3,585 m; geometric parameters of the bus – length – 12,000 m, width – 2,460 m, height – 3,585 m; geometric parameters of trailer links: length – 7,000 m, width – 2,460 m; front wheel track – 1,850 m; bus rear wheel track – 1,800 m; 1,850 m – for trailers; bus spring track: front wheels – 0,75 m; rear wheels – 1,250 m; trailer links – 1,2 m; rigidity of suspension of the front wheels of the bus $C_{p1} = 320$ kN/m, rear wheels – 480 kN/m, trailer wheels – 430 kN/m; type and size of tractor vehicle and semi-trailer tyres: 245/75R17.5; static wheel radius – 0,525 m, rigidity 1250 kN/m; $hg = 1.53$ m [10].

Figure 4, shows the results of the calculation of the bus body roll and trailer when driving in a circle and Figure 5 – the values of the buses wheels loaded and unloaded during the same manoeuvre.

As can be seen from the above dependencies, the roundabouts of a swing-jointed bus show that the body roll of the bus and its side loads are higher than those of the trailer. This is due to the lower position of the centre of mass of the trailer ($hg = 1.21$ m) compared to the bus ($hg = 1.53$ m).

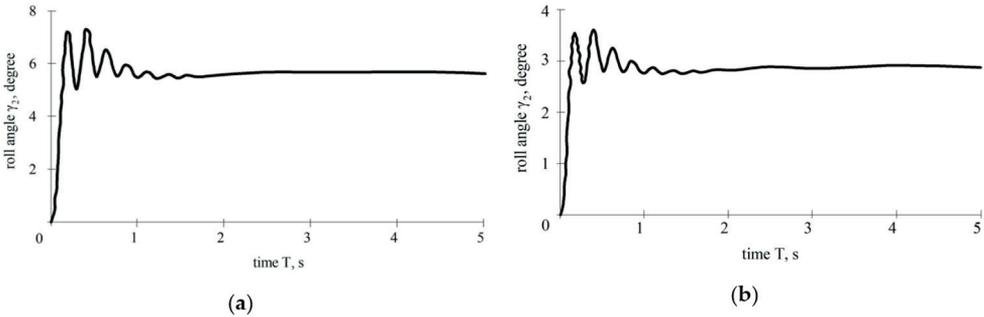


Fig. 4. Angle variation of the buses body lean (a) and trailers (b) in roundabout motion, $R=25$ m, $v=10$ m/s

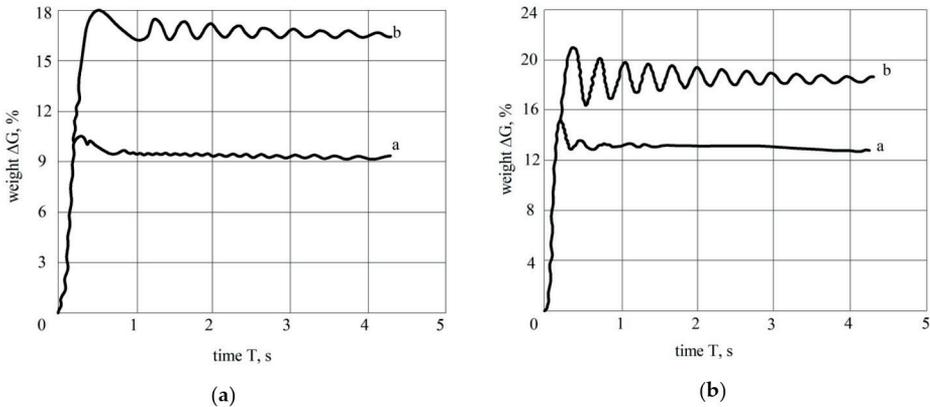


Fig. 5. Refilling the wheels of an outside bus when a road train is in circular motion, taking into account the heel (a) and the effects of the longitudinal and lateral reactions (b): $R = 25$ m; $v = 10$ m/s

The resulting roll angles and bearing angles are based on the calculation of the correction of the wheel withdrawal resistance coefficients of a buses and trailers in order to further calculate the parameters of stationary motions, the critical speed of straight motion and the turning radius of the metrobus.

According to [6] fixed motion of road train on the road plane is straight and circular. In order to study the stability of any of them by traditional methods, it is necessary to construct an equation of disturbed motion, and in non-critical cases, by A.M. Lyapunov, we can confine ourselves to linear approximation analysis. The characteristic equation has a sixth order with very cumbersome coefficients, which makes direct analysis of the equation and analytical results very difficult. Therefore, we will take advantage of the technique that is widely used in wheel stability analysis. It is based on an analysis of the movement of all points of the road train along paths of sufficiently large radii of curva-

ture and a subsequent limit transition in order to obtain the necessary condition for the stability of straight motion and to ensure a circular movement.

Take $v = \text{const}$, $\theta_{1,2,3} = \text{const}$, $M_{C1,2,3} = 0$. If the angles $\theta_1, \theta_2, \theta_3$ have small values, then the same small values will be the generalizing coordinates $u, \omega, \varphi_1, \varphi_2$ [6], meaning that the desired solution will be in a fairly small neighborhood of the origin of the space $\{u, \omega, \varphi_1, \varphi_2\}$. In this case, equation (1) can be linearized in the vicinity of the point $(0, 0, 0, 0)$. These equations take the form:

$$(m + m_1 + m_2)v\omega = Y_1 + Y_2 + Y_3 + Y_4;$$

$$(m_1 + m_2)cv\omega = -aY_1 + bY_2 + c(Y_3 + Y_4);$$

$$(m_1d_1 + m_2l_1)v\omega = (b_1 + d_1)Y_3 + l_1Y_4;$$

$$m_2d_2v\omega = (d_2 + b_2)Y_4 \quad (10)$$

Similarly, [6] of the first two equations of the system, one can obtain the following equation: $mv\omega = (c+a_1)Y_1 + (c-b)Y_2$, which contains only bus (lead) parameters, and is therefore also valid for the three-link metrobus.

In a linear approximation:

$$Y_{nm} = k_{nm}\delta_n \quad (11)$$

where

$$\delta_1 = \theta_1 - \frac{u+a_1\omega}{v}; \delta_2 = \frac{-u-b\omega}{v}; \delta_3 = -\theta_1 - \varphi_1 - \frac{u}{v} + \frac{c+d_1+b_1}{v}\omega;$$

$$\delta_{4S} = -\theta_3 - \varphi_1 - \varphi_2 - \frac{u}{v} + \frac{c+l_1+d_2+b_2}{v}\omega,$$

where k_{nm} – steering resistance coefficient of individual axes of a road train.

Write the system (10) as:

$$\begin{cases} a_{11}u + a_{12}\omega + a_{13}\varphi_1 + a_{14}\varphi_2 = q_1 \\ a_{21}u + a_{22}\omega + a_{23}\varphi_1 + a_{24}\varphi_2 = q_2 \\ a_{31}u + a_{32}\omega + a_{33}\varphi_1 + a_{34}\varphi_2 = q_3 \\ a_{41}u + a_{42}\omega + a_{43}\varphi_1 + a_{44}\varphi_2 = q_4 \end{cases} \quad (12)$$

where

$$a_{11} = \frac{k_1+k_2+k_3+k_4}{v};$$

$$a_{12} = (m + m_1 + m_2)v + \frac{1}{v}[M_1 - M_2 - c(k_3 + k_4) - (k_3d_1 + M_3) - k_4l_1 - (k_1d_2 + M_4)];$$

$$a_{13} = k_3 + k_4; a_{14} = k_4; a_{21} = \frac{1}{v}[c(k_1 + k_2) + M_1 - M_2];$$

$$a_{22} = mcv + \frac{1}{v}[c(M_1 - M_2) + \mu_1 + \mu_2]; a_{23} = k_3d_1 + k_3l_1; a_{24} = k_3l_1;$$

$$a_{31} = \frac{1}{v}c(k_3d_1 + M_3 + k_4l_1); a_{33} = k_3d_1 + M_3 + k_3l_1; a_{34} = k_4l_1;$$

$$a_{32} = (m_1d_1 + m_2l_1)v - \frac{1}{v}[(k_3d_1 + M_3)(c + d_1) + d_1M_3 + \mu_3 + l_1(c + l_1)k_4 + l_1(k_4d_2 + M_4)];$$

$$\begin{aligned}
 a_{41} &= \frac{1}{v}(k_4 d_2 + M_4); a_{42} = m_2 d_2 v - \frac{1}{v}[(k_4 d_2 + M_4)(c + l_1) + d_1 M_3 + \mu_4 + d_2(k_4 d_2 + M_4)]; \\
 a_{43} &= a_{44} = k_4 d_2 + M_4; q_1 = \theta_1 - \theta_2 - \theta_3; q_2 = c\theta_1 + G_2; q_3 = d_1 \theta_3 - G_3 - l_1 \theta_3; \\
 q_4 &= -d_2 \theta_3 - G_4; M_1 = k_1 a; M_2 = k_2 b; M_3 = k_3 b_1; M_4 = k_4 b_2; \mu_1 = k_1 a^2; \mu_2 = k_2 b^2; \mu_3 = k_3 b_1^2; \\
 \mu_4 &= k_4 b_2^2; \theta_1 = k_1 \theta; \theta_2 = k_3 \theta_2; \theta_3 = k_4 \theta_3; G_2 = k_1 a \theta_1; G_3 = k_3 b_1 \theta_2; \\
 G_4 &= k_4 b_2 \theta_3
 \end{aligned} \tag{13}$$

The solution to the system of equations (12) will be the following dependencies [11]:

$$\begin{aligned}
 u &= \frac{\Delta u}{\Delta} = \frac{v\alpha + \frac{1}{v}\beta}{\xi v^2 + \eta} v = v \frac{v^2 \alpha + \beta}{\xi v^2 + \eta}; \omega = \frac{\Delta \omega}{\Delta} = \frac{\frac{1}{v}\gamma}{\xi v^2 + \eta} v^2 = v \frac{\gamma v}{\xi v^2 + \eta}; \varphi_1 = \frac{\Delta \varphi_1}{\Delta} = \frac{\sigma + \psi \frac{1}{v^2}}{\xi v^2 + \eta} v^2 = v \frac{\sigma v^2 + \psi}{\xi v^2 + \eta}; \\
 \varphi_2 &= \frac{\Delta \varphi_2}{\Delta} = -\theta_{2S} - \varphi_1 - \frac{u}{v} + \frac{(k_4 d_2 + M_4)(c + l_1 + d_2 + b_{2S}) + m_2 d_1 v^2}{(k_4 d_2 + M_4)v}
 \end{aligned} \tag{14}$$

Written expressions are accepted:

$$\xi = -m(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times (M_1 - M_2) + [(k_1 + k_2)c + M_1 - M_2] \times [m_2 M_4 (k_3 c_1 - M_3) - m_1 M_3 (k_4 d_2 + M_4)];$$

$$\eta = [c(k_1 + k_2 + M_1 - M_2) \times (k_4 d_2 + M_4) \times \{M_3(M_3 + M_4) - k_3(\mu_3 + M_4 l_1)\} + k_4(k_3 l_1 - M_3) \times (d_2 M_4 + \mu_4)] + [(k_1 + k_2) \times (\mu_1 + \mu_2) - (M_1 - M_2)^2 \times (k_3 d_1 + M_3) \times (k_4 d_2 + M_4)];$$

$$\alpha = -mc[(k_4 d_2 + M_4) \times (\theta_3 M_3 - k_3 G_3) - (k_3 c_1 - M_3) \times (\theta_4 M_4 - k_4 G_4)] - mG_1(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) - (\theta_1 c + G_1)[m_1 M_3 (k_4 d_2 + M_4) - m_2 M_4 (k_3 c_1 - M_3)];$$

$$\beta = (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times [\theta_1(\mu_1 - \mu_2) - G_1(M_1 - M_2) + \mu_1 + \mu_2] \times [(\theta_3 M_3 - k_3 G_3) \times (d_2 k_4 + M_4) - (\theta_4 M_4 - k_4 G_4) \times (k_3 c_1 - M_3) - (\theta_1 c + G_1) \times (k_4 d_2 - M_4) \times (k_3 \mu_3 - M_3^2) + (k_3 l_1 - M_3) \times (M_4^2 - k_4 \mu_4)];$$

$$\sigma = (c\theta_1 + G_1)[m_1 M_3 (k_3 d_2 + M_4) - m_2 M_4 (c_1 k_3 - M_3)] + [(M_1 - M_2)\theta_1 - (k_1 + k_2)G_1] \times [m_1 d_1 (k_4 d_2 + M_4) + M_2 l_1 M_4] + [(k_1 + k_2)c - M_1 - M_2] \times \{m_1 [G_3 (k_4 d_2 + M_4) - c_1 (k_4 G_4 - M_4 \theta_4)] - m_2 M_4 (c_1 \theta_3 - G_3)\} + G_1 m (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) + mc[(k_4 G_4 - \theta_4 M_4) \times (c_1 k_3 - M_3) - (k_3 G_3 - M_3 \theta_3) \times (k_4 d_2 + M_4) \times (d_1 \theta_3 + G_3) - (k_4 G_4 - \theta_4 M_4) l_1];$$

$$\psi = (c\theta_1 + G_1) \times (M_1 - M_2) \times (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) + [G_1(k_1 + k_2) - (M_1 - M_2)\theta_1] \times [(c + l_1 + b_{1\rho}) \times (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) + (M_1 - M_2)^2 - (\mu_1 + \mu_2) \times (k_1 + k_2)] \times [(d_1 \theta_3 + G_3) \times (k_4 d_2 + M_4) - l_1 (k_4 G_4 - M_4 \theta_4)] + (k_4 G_4 - M_4 \theta_4) \times (c_1 k_3 - M_3) - (k_3 G_3 - M_3 \theta_3) \times (k_4 d_2 + M_4) \times \{[(k_1 + k_2)c + M_1 - M_2] \times (c + l_1 + b_{1\rho}) + [c(M_1 - M_2) + \mu_1 + \mu_2]\};$$

$$\gamma = [G_1(k_1 + k_2) - (M_1 - M_2)\theta_1] \times (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) - [c(k_1 + k_2) + M_1 - M_2] \times [(k_3 c_1 - M_3) \times (\theta_4 M_4 - k_4 G_4) - (k_4 d_2 + M_4) \times (\theta_4 M_3 - k_3 G_3)] \tag{15}$$

Even though the stationary motion of a road train is not only straight-line modes, but also circular ones, the condition $R > 0$ must be fulfilled for such movement to be realized. The radius of rotation in the theory of wheeled cars is commonly referred to as the curvature radius $R = v/\omega$ of the longitudinal axis of the tractor vehicle (driving link) whose velocity is directed along the axis [6]. For the radius R according to (14) we have:

$$R = \frac{-[m(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times (M_1 - M_2) + [c(k_1 + k_2) + M_1 - M_2] \times [m_1 M_3 (k_4 d_2 + M_4) - m_2 M_4 (k_3 c_1 - M_3)]] v^2 + \rightarrow}{[(k_1 + k_2) G_1 - (M_1 - M_2) \theta_1] \times (k_3 d_1 + M_3) \times (k_4 d_2 + M_4) - [c(k_1 + k_2) + M_1 - M_2] \times \rightarrow} \\ + \frac{[c(k_1 + k_2) + M_1 - M_2] \times \{(k_4 d_2 + M_4) \times [M_3 (M_3 + M_4) - k_3 (\mu_3 + c_1 M_1)] + k_4 (k_3 c_1 - M_3) \times (d_2 M_4 - k_4 G_4)\} + \rightarrow}{\times [(k_3 c_1 - M_3) \times (\theta_4 M_4 - k_4 G_4) - (k_4 d_2 + M_4) \times \rightarrow} \\ + \frac{(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times [(k_1 + k_2) \times (\mu_1 + \mu_2) - (M_1 - M_2)^2]}{\times (\theta_3 M_3 - k_3 G_3)} \quad (16)$$

As can be seen from (16), the rotational radius of the three-link metrobus depends on its mass and geometrical parameters, the steering resistance coefficients of the axles of the first k_3 and the second trailer link k_4 and the rotation angles of their axes θ_2 and θ_3 .

Let the formula (16) be:

$$R = \frac{l}{\theta} - v^2 \varphi(k_{1i}, k_{2j}, k_{3\rho}, k_{4s}, \theta_1, \theta_2, \theta_3, m, m_1, m_2, a_i, b_j, c, c_1, b_{1\rho}, b_{2s}, L_1, L_2) \quad (17)$$

Since $\varphi(\infty, \infty, \infty, \infty, \theta_1, \theta_2, \theta_3, m \dots L_2) = 0$, the composition of (17) has, as a special case, a linearized expression $R = l/\theta$ for the radius of curvature of the rear axle of the bus on the side-facing rigid wheels. If $\varphi(\infty, \infty, \infty, \infty, \theta_1, \theta_2, \theta_3, m \dots L_2) < \text{or} > 0$, both the R will have more or less than 0 and the three-link metrobus will have insufficient or excessive turning capacity. If $\varphi(\infty, \infty, \infty, \infty, \theta_1, \theta_2, \theta_3, m \dots L_2) = 0$ the three-link metrobus is neutral with respect to turning capacity: its turning radius is the same as that of the metrobus on the side rigid wheels.

When $\theta_1 > 0$, $\theta_2 > 0$, $\theta_3 > 0$, the denominator of the expression (16) is greater than 0, respectively the condition of realization of circular motion and stability of straight motion is met only under $v < v_{cr}$. From (14) we get:

$$\xi v^2 + \eta > 0; \eta > -\xi v^2; v^2 < \frac{\eta}{-\xi} \Rightarrow v_{cr}^2 = \frac{\eta}{-\xi},$$

where

$$v_{cr}^2 = \frac{c(k_1 + k_2) + M_1 - M_2 \{ (k_4 d_2 + M_4) \times [M_3 (M_3 + M_4) - k_3 (\mu_3 + c_1 M_1)] + k_4 (k_3 c_1 - M_3) \times (d_2 M_4 + \mu_4) \} + \rightarrow}{m(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times (M_1 - M_2) + \rightarrow} \\ + \frac{(k_3 d_1 + M_3) \times (k_4 d_2 + M_4) \times [(k_1 + k_2) \times (\mu_1 + \mu_2) - (M_1 - M_2)^2]}{+ [c(k_1 + k_2) + M_1 - M_2] \times [m_1 M_3 (k_4 d_2 + M_4) - m_2 M_4 (k_3 c_1 - M_3)]} \quad (18)$$

According to the expression (18), the critical straight speed of the three-link metrobus was calculated at 32.06 m/s, and this speed is independent of the steering wheel and trailer links corners ($\theta_1 = 0$, $\theta_2 = 0$, $\theta_3 = 0$).

As shown earlier, the interaction forces in the coupling and traction coupling devices do not affect the distribution of loads on the sides of the links of the road train. Therefore a rather complex system – the three-link metrobus can be considered as three systems – the bus, the first trailer and the second trailer that is rolling independently. The roll axis of each link is also considered to be parallel to the support surface, and the movement of the links of the road train in the vertical plane at the angles of halopilation (tangage, trim) and roll are considered to affect lateral motion, primarily and mainly by changing vertical wheel loads, thereby changing the vertical reactions of the supporting surface. In accordance with this concept, a distinction has been drawn between

lateral and longitudinal motion. To this end, the system of equations describing the plane-parallel motion and motion of metrobus links in the vertical plane at the angles of halocaperation (tangage, trim) and roll were integrated separately by Maple software.

The integration of a system of equations describing a three-link metrobus in the vertical plane, together with the equations describing a plane-parallel motion, makes it possible to investigate the behavior of the variables ψ_i , γ_i , and the lateral acceleration and angular yaw velocity of the typical manoeuvres, such as «steering wheel jerk» and «shuffle», $S_1 = 24$ m, Figures 6 to 9.

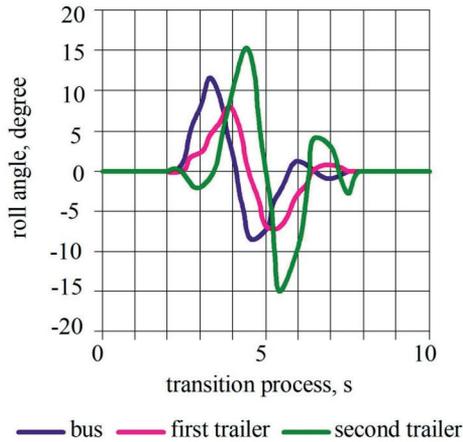


Fig. 6. Changing the roll angle of metrobus links during the transitional time process, $v = 5$ m/s

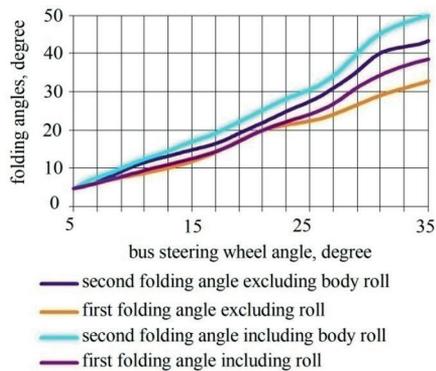


Fig. 7. Dependence of the folding angles the metrobus links on the steering angle of the bus wheels

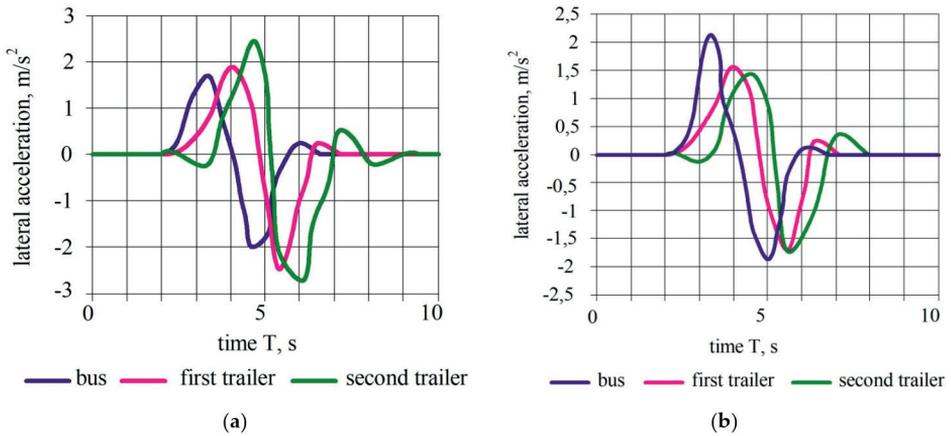


Fig. 8. Modification of the lateral acceleration of metrobus links during the manoeuvres «steering wheel jerk» (a) and «shuffle» (b) at the metrobus speed 5 m/s

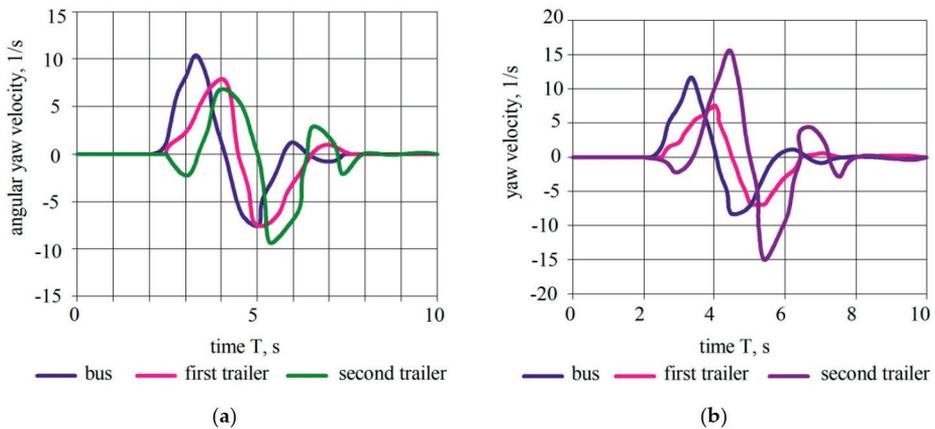


Fig. 9. Variation of the yaw velocity of metrobus links when performing manoeuvres «steering wheel jerk» (a) and «shuffle» (b) at metrobus speed 5 m/s.

As shown in the Figure 6, the biggest roll and loads of the metrobus axles are those of the last trailer, which is the limiting factor in the value of v_{cr} . The angle of rotation of the driven wheels of the tractor unit leads to a change in the direction of movement of the metrobus and, accordingly, to the appearance of folding angles of individual units. Figure 7 shows the reliance between the change in the first (between the longitudinal axes of the bus and the first trailer) and the second angle (between the longitudinal axes of the first and second trailer) of folding of the metrobus links. The analysis of the above dependencies shows that as the steering wheel angle of the bus increases, the difference in the folding angles of the bus increases, with the second folding angle being much greater than the first, especially when the roll of the metrobus body is taken into account.

Lateral accelerations of individual metrobus links and their yaw velocity when performing the manoeuvre «steering wheel jerk» and «shuffle», Figure 8 and 9 show that both the bus and the second trailer are a limiting factor for the various manoeuvres, but that the accelerations do not exceed the permissible 0.4 g (acceleration of gravity).

Figure 10 summarizes the acceleration coefficient of the second trailer as a limiting factor at a speed of 12 m/s and the manoeuvre of the «steering wheel jerk».

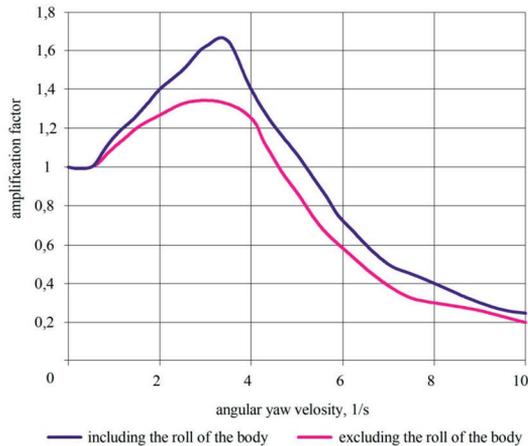


Fig. 10. Correlation of the coefficient of increase of lateral acceleration of the metrobus body with the angle yaw velocity

The analysis of the above graphs also shows that the roll of the body of the metrobus significantly affects the stability of its movement when it performs various manoeuvres. For example, the coefficient of increase of lateral acceleration of the metrobus body, if the roll of the body is taken into account, is increased by 22.2% in comparison with its absence, and this must be taken into account in the analysis of the structure of the metrobus, in particular the running part and the suspension.

3. Conclusions

It has been established that the stability of the metrobus in the performance of typical manoeuvres such as «steering wheel jerk» and «shuffle» it is useful to define by integrating the system of equations, describing the three-link metrobus in the vertical plane together with the equations describing its plane-parallel motion. Under this condition, the folding angles of the metrobus links are defined.

It has been shown that as the steering wheel angle of the bus increases, the difference in the folding angles of the bus also increases, with the second folding angle being significantly greater than the first, especially considering the roll of the metrobus body.

It has been established that the greatest rolls and loads of the metrobus axles are those of the last trailer, which is the limiting factor for the critical speed. For example, the coefficient of increase of lateral acceleration of the metrobus body, if the roll of the body is taken into account, is increased by 22.2% in comparison with its absence, and this must be taken into account in the analysis of the structure of the metrobus, in particular the running part and the suspension.

The lateral accelerations of individual metrobus links and their yaw velocity when performing the manoeuvre «steering wheel jerk» and «shuffle», show that both the bus and the second trailer link are a limiting factor when performing various manoeuvres, but the acceleration value does not exceed the permissible value of 0.4 g (acceleration of gravity), that is, its stability under these conditions is ensured.

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