

THE INFLUENCE OF SHOCK ABSORBER CHARACTERISTICS' NONLINEARITIES ON SUSPENSION FREQUENCY RESPONSE FUNCTION ESTIMATION AND POSSIBILITIES OF SIMPLIFIED CHARACTERISTICS MODELLING

GRZEGORZ ŚLASKI¹, ZBYSZKO KLOCKIEWICZ²

Abstract

The paper shortly presents shock absorber design evolution and resulting achieved characteristics. The way in which suspension performance is evaluated is described giving information about models used for suspension parameter tuning during simulation testing of suspension transmissibility [FRF – Frequency Response Function] for most important suspension assessment criteria. More detailed information about models of shock absorber [damper] nonlinearities of characteristics allows for description of methods of linear and nonlinear suspension models FRF estimation. Testing linear suspension model is possible with the use of analytical transfer function formulas which were used to verify methods for estimation FRF using estimated power spectral density functions of excitation and response signals. Designing appropriate input signal allowing to get useful response signals was necessary for the success of this research. Proposed FRF estimation method was used for linear estimation of nonlinear suspension for a given range of working conditions. It was demonstrated that there is no single value of a damping coefficient which would make the linear model responses similar to the responses of the nonlinear one. Then the bilinear model was proposed, giving good damper static nonlinear characteristic.

Keywords: shock absorber; damper; frequency response function estimation; nonlinear damper model; bilinear damper model

¹ Faculty of Mechanical Engineering, Poznań University of Technology, Piotrowo 3, 60-965 Poznań, Poland, e-mail: grzegorz@slaski.eu, ORCID: 0000-0002-6011-6625

² Faculty of Mechanical Engineering, Poznań University of Technology, Piotrowo 3, 60-965 Poznań, Poland, e-mail: zbyszko.klockiewicz@put.poznan.pl, ORCID: 0000-0003-4353-550X

1. Shock absorbers and their characteristics evolution

Early suspension designs, until the end of the first decade of the 20th century [4], did not use special mechanisms to dampen the vibrations of sprung and unsprung masses of the suspension. At the beginning of the 20th century, mechanical – friction disk shock absorbers were developed that used the friction between the washers and the two levers on their axis of rotation. Friction shock absorbers were commonly used in motor vehicles in the period from around 1910 to the mid-1920s [4].

From 1925, hydraulic shock absorbers were used, initially of a simple design, which were then gradually improved. In the functional sense, their design focused more and more on conscious shaping of the characteristics of the shock absorbers – from the initially used solutions with a constant damping coefficient to the use of asymmetrical and non-linear characteristics – Figure 1.

At the beginning of the use of hydraulic shock absorbers, both double-acting and single-acting structures were used – in the latter case only for the rebound movement [21]. The essence of operation and the construction of shock absorbers did not change from the initial solutions until those from the 1980s.

From the point of view of design solutions, lever shock absorbers were popular for a long time. There were several types – they could have an internal vane, two-vane, single-piston or twin-piston structure [13]. In the 1950s, telescopic shock absorbers gained popularity (patent 1930, production from 1935) in the form of two-tube shock absorbers, and later also one-tube shock absorbers (production from 1950) [4, 20]. Among these solutions, there are also shock absorbers with adjustable characteristics controlled mechanically or electromechanically [4, 13]. One-tube and two-tube telescopic shock absorbers are also currently most often used in the suspensions of modern cars.

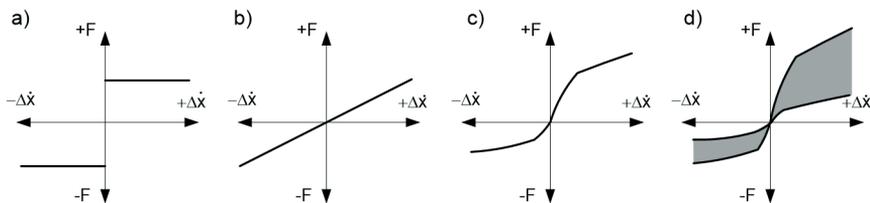


Fig. 1. Simplified characteristics of shock absorbers for their main stages of development of the possibility of shaping damping characteristics: a) friction shock absorber, b) hydraulic shock absorber with symmetrical linear characteristics, c) shock absorber with non-linear asymmetrical characteristics, d) a damper with adjustable characteristics

At the turn of the 20th and 21st centuries, the production of shock absorbers, in which the damping force can be set using either electronically controlled valves or coils generating an electromagnetic field cooperating with magnetorheological fluids [4, 6, 24] was started.

2. Performance evaluation of the vertical vehicle dynamics

Car suspensions have to perform tasks in two main areas – one responsible for vehicle vertical dynamics and second, acting as a mechanism linking wheels to the vehicle body, allowing relative movement and transferring longitudinal and lateral forces between wheels and vehicle body. In the area of vehicle vertical dynamics suspension forms an oscillatory system, affected by kinematic excitation z_r caused by the road surface profile (Figure 2) and also by the force excitations caused by inertial forces acting on vehicle body during manoeuvres in longitudinal or lateral directions.

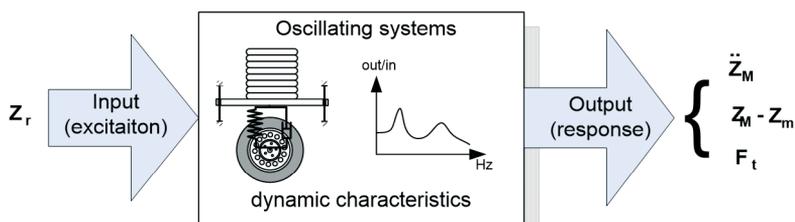


Fig. 2. A car suspension as an oscillatory system with its dynamic characteristics

The kinematic excitations processed by suspension dynamic structure produces such responses as bounce displacement of sprung mass z_M and unsprung mass z_m , the relative displacement of both masses – the suspension deflection [rattle space] $z_M - z_m$, the acceleration of sprung and unsprung masses (\ddot{z}_M, \ddot{z}_m) as well as forces F_t of tire-road contact. The relations between road excitation and listed suspension responses, in a function of excitation frequency, are called the dynamic characteristics of suspension or suspension transmissibility functions [7]. The frequency response function or magnitude–frequency characteristics (Bode magnitude plot) names are also used [9, 15, 17].

The shape and values of frequency responses are very useful to assess suspension performance in regard to comfort, safety and technical limitations concerning limited suspension travel space criteria. Ride comfort can be assessed using sprung mass acceleration transfer (or amplification) function $\ddot{z}_M(\omega)$. The safety potential can be assessed with an analysis of dynamic wheel load amplification function $F_t(\omega)$ and also by analysis of wheel rattle space amplification function. This function is important also from the point of view of kinematic performance of suspension – possible changes of wheel camber and steer angles – and the vertical dynamics influence on lateral dynamics.

These functions can be shaped by tuning parameters of suspension components – sprung and unsprung masses, spring characteristics and shock absorber (damper) characteristics and tire stiffness and damping coefficients or characteristics.

3. Vehicle vertical dynamics models

Tuning suspension Frequency Response Functions shape is often performed using mathematical modelling and simulation. For this purpose vehicle vertical dynamics models, often called vehicle suspension models for short are built in multiple ways with varying degree of complication. One of the most frequently used is a quite simple linear quarter-car model [one car corner suspension model – Figure 3], which was present both in older [9, 15] as well as contemporary works [2, 5]. With increasing ease of use of computer software, more and more research and simulations are done with the use of non-linear models, which is especially noticeable in the last decade [15, 23].

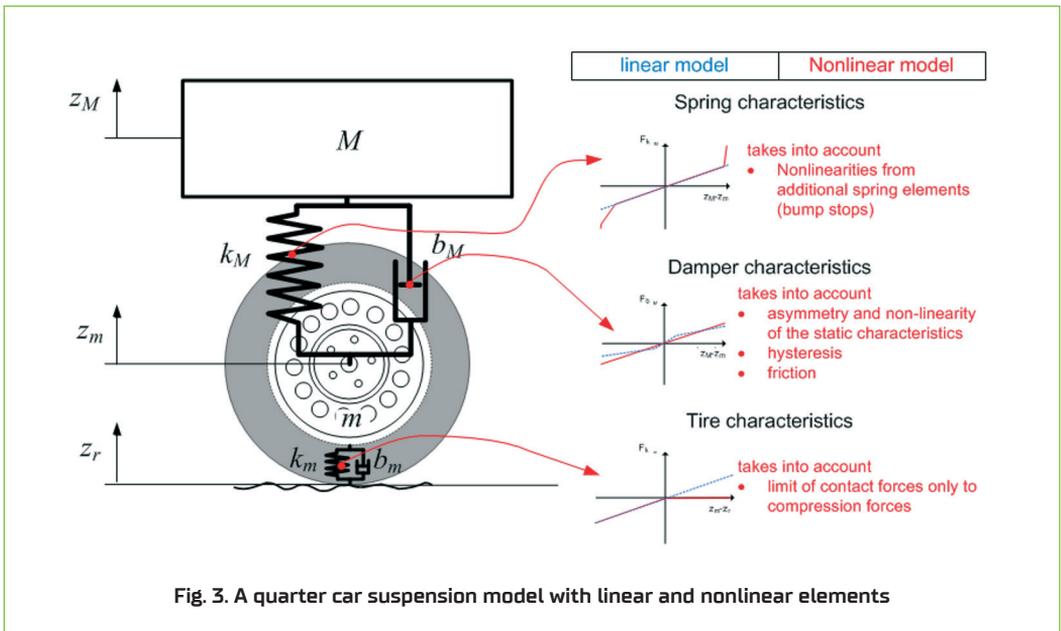


Fig. 3. A quarter car suspension model with linear and nonlinear elements

The type of model which should be used (linear or non-linear) depends on conditions of vehicle operation. Roads of classes [according to ISO 8608] A and B rarely cause suspension to work in the non-linear range of spring and damper characteristics for standard passenger vehicles. An exemplary exception could be a situation in which the vehicle is heavily loaded and excited by B class road. For worse quality roads, especially class D, linear models can yield somewhat satisfactory and realistic results only for very low speeds and an almost empty vehicle. Otherwise, the suspension will regularly enter non-linear working range.

Both the linear and nonlinear models use the same structure of suspension system, the differences lie in the definitions of characteristics of spring and damping elements. The nonlinear tire model can limit tire forces only to compression force (zero tire force when there is no road-tire contact). Nonlinear spring characteristics means that big force increases when suspension enters the range of bump stop work is taken into account. Nonlinear damper characteristic is even more complicated because of asymmetry and nonlinearity of its static

characteristics, hysteresis of damping force and friction as well. In that case it is necessary to build shock absorber forces model instead of interpolating its characteristics.

4. Shock absorber characteristics nonlinearities and their modelling

An example of a modern adjustable shock absorber characteristics is presented in a Figure 4, showing asymmetry, nonlinearity and hysteresis [22]. Shock absorbers models used for testing vehicle dynamics establish a relationship between the damping force and damper compression and rebound velocity.

The main component of such models – the static characteristic of damping force of shock absorber – has to model main relationship between damper compression and rebound velocity. Modelled force is the average damping force for the given compression/rebound velocity. For nonlinear and asymmetrical characteristics interpolation of experimental or designed characteristics is used as a tool for estimating damping force. For linear models damping coefficient can be used.

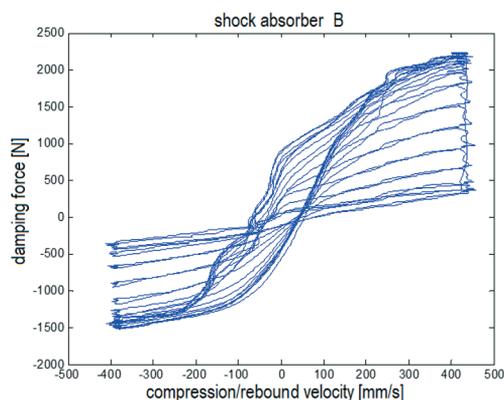


Fig. 4. An example of a modern adjustable shock absorber characteristics [22]

When dynamic behavior for high damping forces and high velocities of damper compression/rebound is important, then the hysteresis needs to be modelled. Models which allow to model appropriate hysteresis loop can be found in literature and were also implemented by authors in this and other research work. Every suspension and shock absorber itself have some amount of dry friction which also needs to be modelled. For adjustable absorbers, the response time is also important and needs to be modelled to appropriately mimic the response delay time of shock absorber to control signal.

5. Quarter car model frequency responses as a measure of suspension performance

Vehicle ride quality assessed by analysis of sprung mass accelerations is influenced by two main factors:

1. the vehicle's suspension system dynamic characteristics;
2. the road kinematic excitation resulting from the pavement surface and vehicle velocity.

In the process of assessing vehicle suspension performance to analyze safety and rattle space limitations also other variables are taken into account – suspension deflection and tire dynamic load.

All of these variables are vehicle suspension responses to input in the form of road excitation. As the road excitation is time frequency signal vehicle suspension transforms it to responses that are also time frequency signals. To analyze how suspension transforms inputs into outputs (dynamics responses) the input and response signals can be expressed in the form of a Power Spectral Density (PSD) function. The relation between response PSD function and the excitation PSD function is defined by the Frequency Response Function (FRF). It is also called transmissibility function. This function acts as a frequency weighting factor between the input and output. Depending on the suspension performance criteria and resulting variable being analyzed, various functions are analyzed:

- sprung mass acceleration transmissibility;
- suspension deflection transmissibility;
- tire load (vertical force) transmissibility [17];
- sprung mass displacement transmissibility [e.g.[8]];
- unsprung mass displacement transmissibility [e.g.[8]].

The interpretation of FRF function can be made easier by conducting simple experiment to determine it. The experiment was about exciting the suspension system with constant amplitude input signal, while slowly changing its frequency. The amplitudes of response signal can be then easily compared with amplitudes of excitation signal. The multiplication factor of amplitudes for every frequency can be calculated. And that values are FRF function values – Figure 3. The example of this method for such exemplary signals of suspension deflection frequency in response to 0.01 m amplitude road excitation input signal is presented in Figure 5. Two resonances are clearly visible – for sprung mass and for unsprung mass. At input signal frequency near 0 Hz suspension almost does not move. Also at frequencies much higher than unsprung mass resonance there is very little suspension deflection.

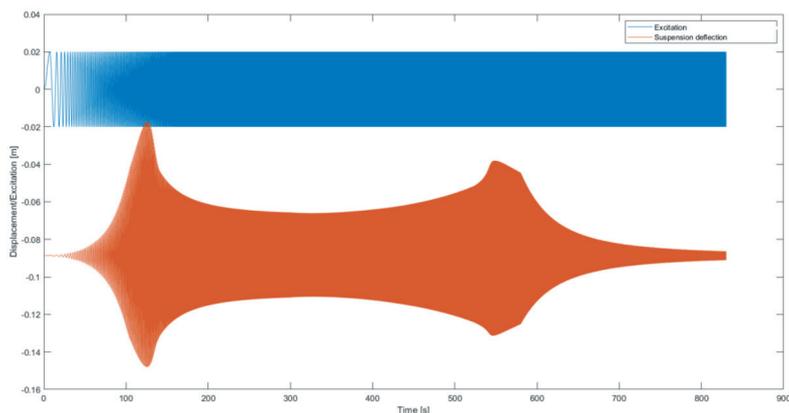


Fig. 5. Excitation and response signal (suspension deflection) of suspension excited by constant amplitude sinusoidal input signal with linearly increasing frequency

An example of practical significance of FRF is relation between EUSAMA (European Shock Absorbers Manufacturers Association) test results [12] and tire vertical force FRF presented in Figure 6. Changes of tire vertical force are responses to constant amplitude (0.003 m) kinematic excitation with frequency increasing from 0 Hz to 25 Hz. Such input is used on EUSAMA test stands during Periodical Vehicle Inspections to assess suspension technical condition. Yellow line presents the tire vertical force FRF assuming that frequency is linearly changing according to time.

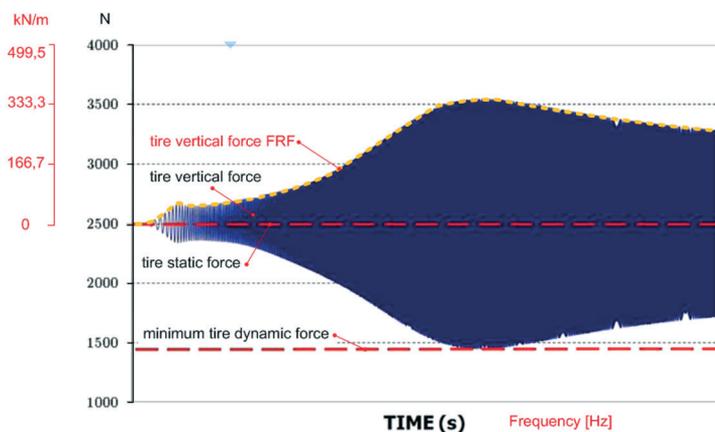


Fig. 6. The EUSAMA test and tire vertical force FRF [red scale and yellow line]

Tire vertical force Frequency Response Function amplitudes in an unsprung mass resonance frequency range can be used to calculate maximum and minimum tire dynamic loads. If it is for example 333.3 kN/m then multiplying 0.003 m excitation we obtain amplitude of dynamic wheel load equal to 999 N. If the static load is equal to 2500 N then we obtain maximum wheel load equal to 3500 N and minimum equal to 1500 N. Calculation of EUSAMA index will give a following result: $EU=1500/2500=0.6$.

6. Frequency response function estimation methods

6.1. Calculation of frequency response function for linear suspension model

In the case of widely used linear models of dynamic systems amplification functions can be evaluated using fundamental methods of control engineering [exemplary use of these methods for a linear quarter car model is described e.g. in [10, 18]].

Calculations are done by determining Fourier transforms of differential motion equations with zero initial conditions and then by determining formulas for frequency response functions for selected inputs and outputs. These functions can be expressed in algebraic and exponential form – formulas [1] and [2] [3]:

$$G(j\omega) = \frac{x(j\omega)}{u(j\omega)} = P(\omega) + jQ(\omega) = A(\omega)e^{j\varphi(\omega)} \quad (1)$$

Gain of frequency response function is expressed as:

$$A(\omega) = \sqrt{P^2 + Q^2}|_{\omega} = \frac{A_{\text{output}}(\omega)}{A_{\text{input}}(\omega)} \quad (2)$$

The physical interpretation is the gain between magnitude of excitation and magnitude of response in the form of sinusoidal signal.

Frequency response function argument φ can be expressed as:

$$\varphi(\omega) = \arctg \frac{Q}{P}|_{\omega} \quad (3)$$

and its physical interpretation is the phase shift of sinusoidal signals – input and output.

6.2. Estimation of frequency response function of nonlinear suspension model and real suspension

Frequency characteristics of suspension can also be estimated by analysing the relations between excitation [input] signal and response [output] signals. These signals can be measured and acquired during tests on a real suspension or simulated with the use of different types of suspension model – for example nonlinear model. The responses of a non-linear

system do not depend on the frequency alone, but on the input amplitude as well. But assuming that one is interested in a typical range of suspension work (for typical road excitation levels) it is possible to assume that the amplitude of response will be proportional to the amplitudes of excitation.

Frequency response evaluation can be made using the estimators of power spectral density and cross power spectral – expression [4] [1]. To calculate these densities it is necessary to use prepared measured or simulated appropriate signals in the form of time histories:

$$\hat{H}_{xy}(\omega) = \frac{\hat{G}_{xy}(\omega)}{\hat{G}_x(\omega)} \quad (4)$$

where:

$\hat{G}_x(\omega)$ – estimator of power spectral density of input signal – kinematic excitation signal,
 $\hat{G}_{xy}(\omega)$ – estimator of cross power spectral density of input and output signals.

The use of formula [4] requires knowledge of power spectral density – PSD (power spectrum value divided by frequency band for which a given PSD value was determined). The two-sided continuous power spectral density can be calculated as a limit [9]:

$$S_x(\omega) = \lim_{\Delta\omega \rightarrow 0} \frac{\Delta\langle f_n^2(t) \rangle}{\Delta\omega} \quad (5)$$

where:

$\Delta\langle f_n^2(t) \rangle$ – power share of function $f(t)$ near the frequency ω_n ,
 $\Delta\omega$ – frequency band for which power share is determined.

Determining power spectrum or power spectrum density exactly is a complex task, which is caused mainly by a limited set of samples [19]. Due to that measurements only allow to estimate the power spectrum or power spectrum density. Various methods can be used to estimate the power spectral density, for example:

- squaring the magnitude of $f(t)$ signal Fourier transform and thus obtaining the periodogram;
- calculating discrete Fourier transform (DFT);
- autocorrelation $f(t)$ signal function [19].

Calculating the estimates of power spectral density with few signal samples gives unsatisfactory results, which are visible as considerable PSD signal fluctuations. To obtain smoothed PSD estimates such methods as calculating and averaging periodograms can be used. One of the most commonly used methods is the Welch method, which produces the so-called modified periodogram. The Welch method is a modified version of the Bartlett method [19].

Both methods divide the signal into several periods, for which shorter periodograms are calculated and next an averaged periodogram over all the segments is calculated.

The Bartlett method uses segments which do not overlap and Welch methods uses overlapping segments and time windows other than rectangular – e.g. the Hanning windows. As a result the Welch method estimates PSD with a smaller variance compared to the Bartlett method. This method is available amongst others in Matlab software and allows to estimate the power spectral density (also cross-spectral density) expressed in units of power per radian per sample (rad/Sa) or Hz.

Using such estimated power spectral density functions it is able to calculate estimate of the frequency response function. It can be done also with the *tfestimate* function in Matlab software, which automates all the process. Authors of the paper tested use of the *tfestimate* function in Matlab software to estimate the frequency response function for linear, bilinear and nonlinear models of quarter car suspension.

7. Test of frequency response function estimation

The test of frequency response function estimate consist of three phases:

1. design of an input signal with characteristics that facilitate obtaining good estimation of Frequency Response Function and comparison it with FRF calculated analytically for fully linear model;
2. estimation of FRF for nonlinear suspension model (nonlinear tire, spring and shock absorber) and its comparison with suspension models with fully linear and symmetrical bilinear shock absorber models and
3. testing the possibility to identify damping coefficients for bilinear shock absorber model that best matches nonlinear model.

7.1. Suspension model used for test

The vehicle model used in simulation was a non-linear quarter-car model with three different shock absorber model variants:

1. non-linear damper;
2. linear damper;
3. bilinear damper.

The tire forces model had a non-linearity as a possibility for a tire to loose contact with the road surface – tire forces to drop to 0 N in that situation. This meant the tire forces would never be negative. The positive tire forces were linearly dependant on the tire deflection and its velocity. The characteristic is presented in Figure 7.

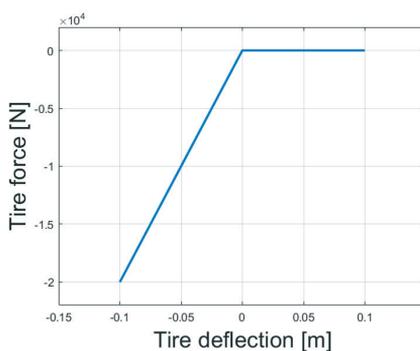


Fig. 7. Tire force model used in test

Other non-linearities in the model were:

- the suspension spring, which used characteristic acquired in tests of a front suspension spring that was a part of a MacPherson strut;
- the rubber bump stop making the characteristic progressive.

The bump stop included an artificially added point when the spring is completely compressed – in which case the force to compress it any further dramatically increases. The full characteristic is presented in Figure 8.

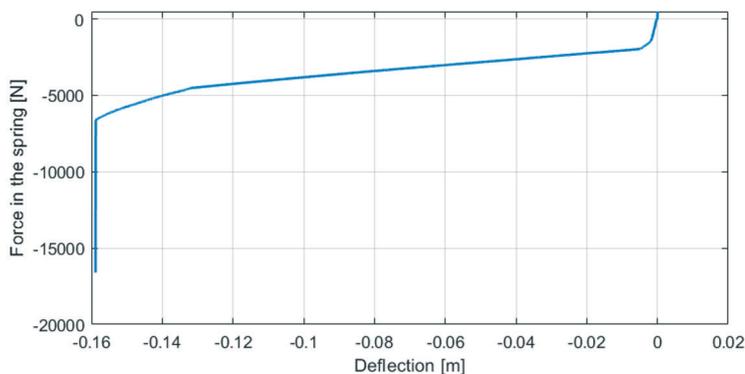


Fig. 8. Spring characteristic used in test

The non-linear version of damper model used the front damper characteristic, which is presented in Figure 9. The model also includes modules simulating hysteresis and internal friction of a damper, which could be disabled and enabled [17].

The use of Matlab **tfestimate** function is not the only condition to get good FRF estimate. The preparation of an appropriate input signal is also necessary. It should represent broad spectrum of frequencies for good transfer function estimation. Authors used the modified version of so called chirp signal which is sinusoidal signal of a linearly swept frequency in a given time. The modification was necessary because the normally generated “chirp” signal (using Matlab or Simulink generator) goes through low frequencies really quickly – it has not even one full cycle in the lowest frequencies while in the high frequencies there are many cycles present. Such a signal will not allow to produce good enough results in FRF estimation.

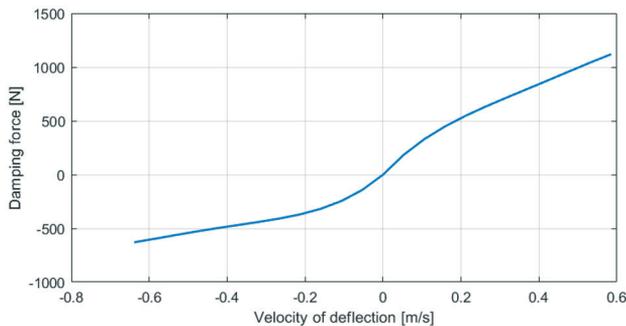


Fig. 9. Damper characteristic of non-linear model used in test

7.2. Test first phase – input signal preparation for FRF estimation

The **tfestimate** function in Matlab needs at least a few cycles (around 10) in or around a given frequency to estimate FRF properly. This forced authors to produce more complicated chirp signal – signal that was combined from three shorter signals lasting enough time to consist of multiple cycles of similar frequencies occurring during simulation:

1. the first one signal for only low frequencies up to 3 Hz changing linearly for the first 300 s;
2. the second signal for medium-range frequencies from 3 Hz to 10 Hz changing linearly from 300 s to 580 s;
3. the last signal for high frequencies from 10 Hz to 35 Hz changing linearly from 300 s to 580 s.

As the suspension and its model has a nonlinear stiffness elements, another very important condition for input signal was to produce response signals in a range of suspension operational deflections. It needed to control the response range. In order to acquire important data about frequencies around 25 Hz, the input signal was prolonged to 35 Hz to avoid distortions for highest frequencies.

7.3. Test first phase – FRF estimation for linear model

To have some reference result for testing method of estimation FRF functions with use of **tfestimate** function the results of analytical calculation of this function was prepared and used for comparison. The magnitude of Frequency Response Function for sprung mass displacement to road kinematic input calculated and estimated with use of Matlab **tfestimate** function is presented in Figure 10.

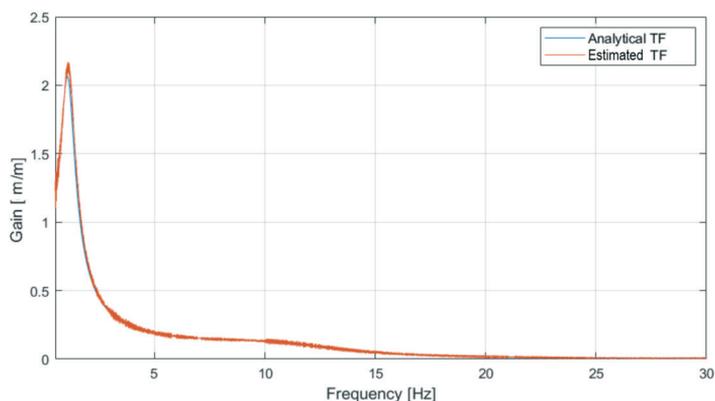


Fig. 10. Estimated and analytically calculated frequency response function for sprung mass displacement

Formula for analytical FRF function calculation is following:

$$H_{z_M}(s) = \frac{(b_M s + k_M) k_m}{m M s^4 + (m b_M + M b_M) s^3 + (M k_M + m k_M + M k_m) s^2 + b_M k_m s + k_m k_M} \quad (6)$$

The estimated FRF [red in Fig. 8] was obtained using Matlab **tfestimate** function and their values for almost 99% of a frequency range are the same as analytical FRF. The only difference is in a range of first resonance – the value of FRF is 5% lower.

7.4. Test second phase – FRF for nonlinear suspension model and comparison with linear and bilinear damper model

In the next phase of tests FRF for five different responses of suspension dynamics were estimated for base nonlinear model of suspension and two other models with different shock absorber models. One model was fully linear damping force model and the second one was symmetric bilinear model with two damping coefficients. One coefficient had a greater value – it was for a range from -0.2 m/s to 0.2 m/s. The second one, for the higher deflection speed values, had a smaller value. The quarter car model responses obtained during simulation were then used to estimate the transfer functions between kinematic excitation and a given suspension response. Estimated FRF functions were following:

- sprung mass displacement;
- unsprung mass displacement;
- sprung mass acceleration;
- suspension deflection;
- cumulative tire force.

It was useful to calculate the probability density of a given suspension deflection to appear, in order to establish whether or not given excitation would cause the suspension to work in a normal operation range.

The first stage of the test was made with fully linear damper model (single damping coefficient), which had its value changed in iterations in order to find the one value that will produce effects that most closely resemble FRF function of the non-linear model FRF. A broad spectrum of damping coefficients were tested – from 1000 Ns/m thru every 200 Ns/m up to 3000 Ns/m. This phase was intended to check what damping coefficient would give results most similar to those of a non-linear model. In this phase it became clearly visible, that there is no single value of a coefficient which would make the linear model responses similar to the responses of non-linear one. The only possibility was to find a coefficient which produced responses similar near the first or the second resonant frequency, but not both at the same time – Figure 11.

In the second stage of testing, previously found damping coefficients' values which gave FRF the most similar to FRF of non-linear models in both frequencies were tested once again. This time iterations were made with much smaller increases of 10 Ns/m per iteration starting from $c_M = 1400$ Ns/m for sprung mass resonance and from $c_M = 2600$ Ns/m for unsprung mass resonance.

Those two values were also used to test the bilinear symmetric model for checking if it is able to approximate Frequency Response Functions better than that with values calculated based on the only static characteristic – not taking into account hysteresis and friction in the damper.

Bilinear symmetric model was proposed as a slightly more complex, yet still easily implementable while eliminating the main disadvantages of linear model. The bilinear symmetric model consist of two linear functions, connected at the joint point (Figure 12). The name bilinear comes from the fact, that there are only two different linear damping coefficients. Although the characteristic consists of three lines, two of them share the same slope. This characteristic is still simplified – for example, it is still symmetrical, while the real shock absorbers characteristic differs depending on whether the damper is in compression or rebound phase.

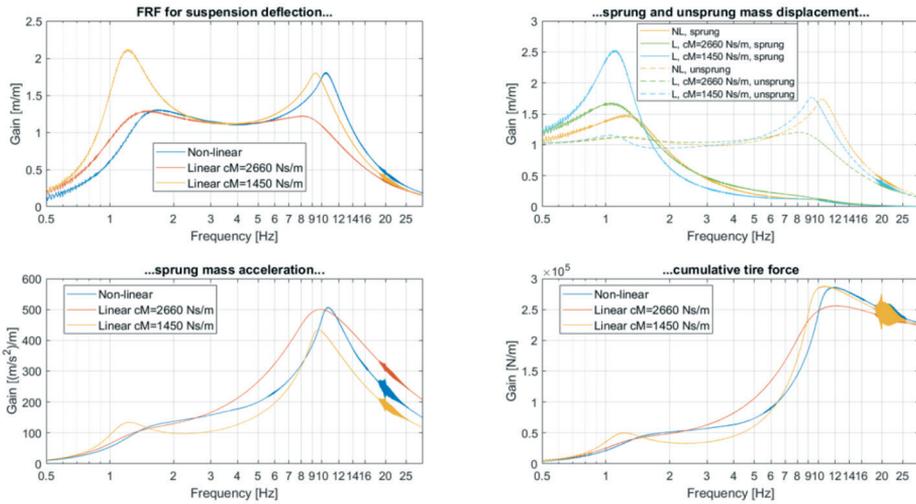


Fig. 11. Comparison of FRF of nonlinear quarter car model and two linear shock absorber models [$b_M=2660$ Ns/m and $b_M=1450$ Ns/m]

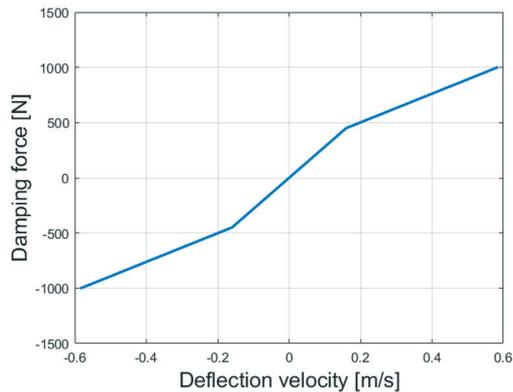


Fig. 12. Bilinear model of shock absorber (damper) characteristics

Best matched Frequency Response Function was obtained for bilinear model with damping parameters $b_{M1}=2800$ Ns/m and $b_{M2}=1220$ Ns/m - as presented in Figure 13. Found coefficients differ from values used in reference nonlinear model, which were determined to be about $b_{M1}=2815$ Ns/m and $b_{M2}=1440$ Ns/m. The difference is caused by the fact that the reference nonlinear shock absorber is also nonsymmetrical.

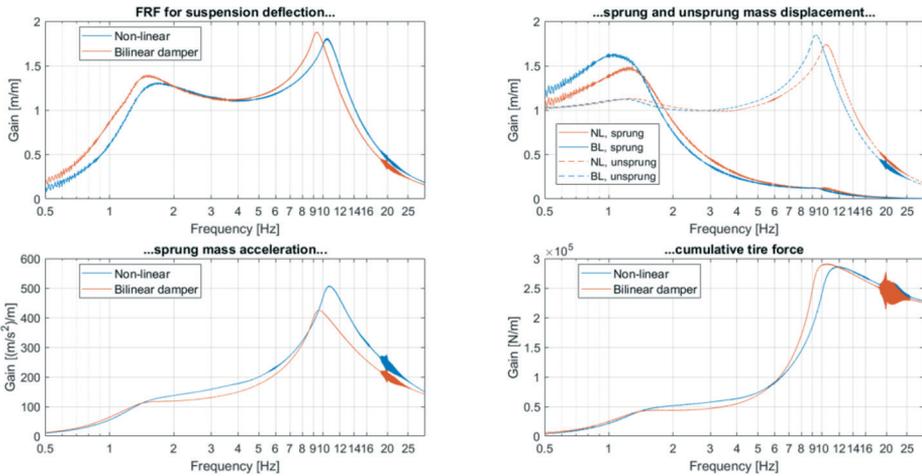


Fig. 13. Best matched FRF of bilinear model compared with FRF of nonlinear model of shock absorbers

The last stage of the test was intended to check which of nonlinear elements of shock absorber model – friction force or hysteresis – makes it impossible to find linear or bilinear model perfectly estimating nonlinear model behavior. During simulation tests Frequency Response Functions for different versions of nonlinear shock absorber model (with friction and hysteresis, without friction, without hysteresis and without friction and hysteresis) and bilinear model were compared – results are presented in the Figure 14. The best match with bilinear model is achieved for nonlinear damper with asymmetrical static characteristics without friction and hysteresis.

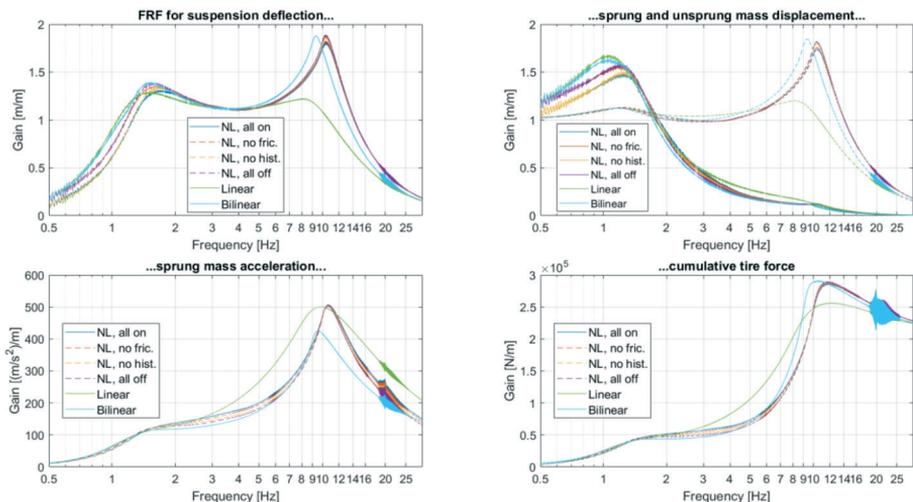


Fig. 14. Results of comparison of FRF for bilinear and nonlinear shock absorber models

Nonlinear model without friction in low frequencies has similar FRF to its version without friction and hysteresis and in a range of higher frequencies more similar to the version with friction and hysteresis.

Nonlinear model without hysteresis in a low frequencies range has FRF more similar to fully nonlinear model (hysteresis and friction enabled) and in a range of higher frequencies has them like the version of a model without friction and hysteresis. These facts allowed researchers to conclude that friction is more important in modelling behavior of suspension in a range of sprung mass resonance frequencies and hysteresis is more important in a range of unsprung mass frequencies.

8. Conclusions

The paper shortly presented shock absorber design evolution and achieved characteristics showing that modern shock absorbers have nonlinear, asymmetric characteristics with friction and hysteresis. As a useful method for suspension performance analysis in a frequency domain, authors proposed to use Frequency Response Function estimation method using power spectral density functions of excitation and response signals.

This method of Frequency Response Function estimation for nonlinear models of quarter car suspension, can be also used for physical car suspensions and measured wheel excitation and suspension responses.

The proposed method is based not only on Frequency Response Function estimation with the use of Power Spectral Density function estimation but also on a special preparation of the input signal modelling kinematic excitation from road profile. Performed tests proved that for linear models it is possible to get almost the same frequency response function shapes using estimation and analytical calculations.

It was demonstrated that for nonlinear models their ideal estimation using linear damper model is impossible. The reasons for this fact are as follows:

- due to nonlinearity of shock absorber characteristics damping coefficient for the sprung mass resonance is much higher than for the unsprung mass resonance;
- due to presence of friction, especially at low frequencies, nonlinear shock absorber with friction has a lower FRF gains due to different friction force behavior than viscous damping forces – essential friction force is present even for the smallest deflection velocities when viscous damping force at the smallest deflection velocities are almost absent;
- due to presence of hysteresis of damping force there is a difference in all testing frequency ranges; it causes the biggest differences in an unsprung mass resonance.

Completed tests demonstrated that use of symmetrical bilinear model of shock absorber can give quite good estimation of fully nonlinear damper in a sense of testing various Frequency Response Function. Obtained results give similar values of gain at the sprung and unsprung masses resonances with small shift in a value of resonance frequency.

Some restriction to presented conclusion is a fact that tests were made only for a linear range of spring characteristic, which means that in order to extend them for other input amplitudes reaching non-linear spring work ranges, it would be necessary to run much more simulations and create a FRF of two variables – both frequency and amplitude.

9. References

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