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PRECRASH VEHICLE VELOCITY DETERMINATION USING INVERSE SYSTEM AND TENSOR PRODUCT OF LEGENDRE POLYNOMIALS - SUBCOMPACT CAR CLASS

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Abstract

Presented paper discusses new approach to EES parameter determination in frontal car crash based on the tensor product of Legendre polynomials. In this paper Subcompact Car Class was analyzed using that method. Data that was used to perform analyses introduced in this paper was taken from National Highway Traffic Safety Administration (NHTSA) database. Such database consists of considerate number of test cases along with various information including vehicle mass, crash velocity, chassis deformation etc. New approach to the problem of determining the EES parameter was necessary due to the low accuracy of the currently used methods. Linear models used up till now for accident reconstruction show significant error as the relationship between mass, velocity and deformation cannot be well approximated with a flat plane. Proposed model produces better results, because of the nonlinear dependence of said parameters. This paper also includes a calculation example presenting a comparison of linear and nonlinear method on an actual crash test.

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1. Introduction

The most popular method used currently for precrash velocity determination based on car deformation is CRASH3. It bases on linear models and comes from 1980s. It is very simple and useful but cause very large error reaching in modern cars up to 30%. During the last 40 years car construction has changed significantly. It has developed from body-on-frame to unibody construction. Also new materials has been put into use in car construction [5], like plastics, or High Strength Steel. It caused that CRASH3 method become outdated, but it is still used due to the lack of new methods [22, 25, 36].

CRASH3 [19, 20, 23] method was created when personal computers were developing and not in common usage. In the past experts had to calculate dissipated energy manually, so it was necessary to use simple methods [2, 18, 24]. Nowadays due to developing computer science experts can perform complicated calculation in a quick way [30, 31, 38]. Taking into consideration all facts mentioned above authors started researching the new methods. Authors decided to base on nonlinear models, that are more complicated but in most cases significantly increase accuracy. In this paper authors present nonlinear method based on tensor product of Legendre polynomials, focusing on a particular vehicle class [15, 28, 37]. Presented method allowed to calculate EES parameter

$$\mathbf{EES} = \sqrt{\frac{\mathbf{2W_{def}}}{\mathbf{m}}} \tag{1}$$

The EES parameter represents the velocity, that is all absorbed to deformation of the vehicle impacting a rigid obstacle [26, 29, 33]. During the impact, there are no elastic deformations, therefore the vehicle's kinetic energy is fully used on the chassis deformation work [11, 32, 35]. EES parameter along with methods that describe dissipation of energy on after crash car movement is used to determine the vehicle velocity right before the impact. Determination of pre-crash velocity is a standard procedure in car accident reconstruction [3, 6, 34] and it is necessary to define it as precisely as possible.

In this paper authors assumed that EES parameter of collision depends on two factors, mass of the vehicle and its deformation ratio. It is not the first approach to determine new nonlinear method. Ideas found in literature concern the inverse system [1, 4]. Those methods determine the magnitude of coefficient $\mathbf{b_k}$, the nonlinear slope. The precrash velocity $\mathbf{V_t}$ depends on the deformation coefficient $\mathbf{C_s}$ [12, 16, 21] through this coefficient. The $\mathbf{C_s}$ coefficient is used to establish the body deformation, which is an arithmetic average of deformation depth in six control points $\mathbf{C_1}$ to $\mathbf{C_6}$ [9, 10, 14].

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The method is based on orthogonal functions, the Legendre polynomials, over the interval of [−1,1]. To apply the Octave software for this approach, Legendre polynomials had to be rescaled and renumbered. Once this is done, the next step of least square approximation can be applied. Authors based their method on data shared by National Highway Traffic Safety Administration (NHTSA) and decided to focused on frontal collisions [13]. Apart from data from actual crash tests, NHTSA developed a few simulation models [7, 8, 27] and is constantly striving to improve road safety and reduce number of casualties [17].

2. Tensor product method description

Firstly, let us assume that there is a set of points $(\mathbf{x_n,y_n,z_n})_{\mathbf{n=1}}^{\mathbf{N}}$ and function family $(\mathbf{h_m})_{\mathbf{m=1}}^{\mathbf{M}}$ (two variables functions). To minimize the expression (2), $(\mathbf{a_m})_{\mathbf{m=1}}^{\mathbf{M}}$ coefficient has to be found.

$$\sum_{n=1}^{N}\left(z_n-\sum_{m=1}^{M}a_m h(x_n,y_n)\right)^2 \tag{2}$$

Then, the least square approximation problem can be reduced down to a linear one:

$$\begin{pmatrix} \sum_{n=1}^{N} h_1(x_n,y_n)h_1(x_n,y_n) & \cdots & \sum_{n=1}^{N} h_1(x_n,y_n)h_M(x_n,y_n) \\ \vdots & \ddots & \vdots \\ \sum_{n=1}^{N} h_M(x_n,y_n)h_1(x_n,y_n) & \cdots & \sum_{n=1}^{N} h_M(x_n,y_n)h_M(x_n,y_n) \end{pmatrix}\begin{pmatrix} a_1 \\ \vdots \\ a_M \end{pmatrix} = \begin{pmatrix} \sum_{n=1}^{N} y_n h_1(x_n) \\ \vdots \\ \sum_{n=1}^{N} y_n h_M(x_n) \end{pmatrix} \tag{3}$$

The family of functions $(\mathbf{h_m})_{\mathbf{n=1}}^{\mathbf{M}}$ will be represented by product tensors of Legendre polynomial. Those are considered to be a sequence of polynomials $(\mathbf{P_m})$ which can be express the following iterative formula:

$$\forall_m \geq 1 (m+1)P_{m+1}(x) = (2m+1)x \cdot P_m(x) - mP_{m-1}(x) \tag{4}$$

Where $\mathbf{P_0}\ (\mathbf{x}) = \mathbf{1}$ and $\mathbf{P_1}\ (\mathbf{x}) = \mathbf{x}$, assuming a range of [−1,1]. The first Legendre polynomials take the form of:

$$P_0(x) = 1,\ P_1(x) = x, P_2(x) = \frac{1}{2}(3x^2-1), P_3(x) = \frac{1}{2}(5x^3-3x), \ldots \tag{5}$$

One of the features of Legendre polynomials is orthogonality:

$$\forall_{i\neq j}\int_{-1}^{1} P_i(x)P_j(x)\,\mathrm{d}x = 0 \tag{6}$$

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It stems from the fact that Legendre polynomials are created though orthogonalization of Gram–Schmidt function family $\{\mathbf{1},\ \mathbf{x},\ \mathbf{x^2},\ \mathbf{x^3},\ ...\}$. This is an useful feature, since matrix $\mathbf{M}$ on left hand side (4) is closer to diagonal matrix. It also assures a smaller $(\mathbf{a_m})_{\mathbf{n=1}}^{\mathbf{M}}$ coefficient error.

Since Legendre polynomials sequence is renumbered, then $\mathbf{Q_m} = \mathbf{P_{m-1}}$. Then following is obtained:

$$\forall_{m\geq 3}(\boldsymbol{m}-\mathbf{1})\boldsymbol{Q_m}(\boldsymbol{x}) = (\mathbf{2}\boldsymbol{m}-\mathbf{3})\boldsymbol{x}\cdot\boldsymbol{Q_{m-1}}(\boldsymbol{x}) - (\boldsymbol{m}-\mathbf{2})\boldsymbol{Q_{m-2}}(\boldsymbol{x}) \quad (7)$$

where $\mathbf{Q_1\ (x) = 1}$ and $\mathbf{Q_2\ (x) = x}$. If an arbitrary interval $[\mathbf{a,b}]$ will be applied, the polynomials need to be rescaled and the following relation can be used:

$$\boldsymbol{f_m}(\boldsymbol{x}) = \boldsymbol{Q_m}\left(\frac{\mathbf{2}\boldsymbol{x}-\boldsymbol{a}-\boldsymbol{b}}{\boldsymbol{b}-\boldsymbol{a}}\right) \quad (8)$$

Tensor product of two function $\mathbf{f}$ and $\mathbf{g}$ can be described as:

$$\boldsymbol{h}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{f}\otimes\boldsymbol{g}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{f}(\boldsymbol{x})\boldsymbol{g}(\boldsymbol{y}) \quad (9)$$

For this approach, the $(\mathbf{f_i})_{\mathbf{i=1}}^{\mathbf{5}}$ and $(\mathbf{g_j})_{\mathbf{j=1}}^{\mathbf{5}}$ constitute the first five Legendre polynomials. This results in 25 tensor products.

$$\begin{array}{ccccc}
\boldsymbol{h_1} = \boldsymbol{f_1}\otimes\boldsymbol{g_1}, & \boldsymbol{h_2} = \boldsymbol{f_1}\otimes\boldsymbol{g_2}, & \boldsymbol{h_3} = \boldsymbol{f_1}\otimes\boldsymbol{g_3}, & \boldsymbol{h_4} = \boldsymbol{f_1}\otimes\boldsymbol{g_4}, & \boldsymbol{h_5} = \boldsymbol{f_1}\otimes\boldsymbol{g_5}, \\
\boldsymbol{h_6} = \boldsymbol{f_2}\otimes\boldsymbol{g_1}, & \boldsymbol{h_7} = \boldsymbol{f_2}\otimes\boldsymbol{g_2}, & \boldsymbol{h_8} = \boldsymbol{f_2}\otimes\boldsymbol{g_3}, & \boldsymbol{h_9} = \boldsymbol{f_2}\otimes\boldsymbol{g_4}, & \boldsymbol{h_{10}} = \boldsymbol{f_2}\otimes\boldsymbol{g_5}, \\
\boldsymbol{h_{11}} = \boldsymbol{f_3}\otimes\boldsymbol{g_1}, & \boldsymbol{h_{12}} = \boldsymbol{f_3}\otimes\boldsymbol{g_2}, & \boldsymbol{h_{13}} = \boldsymbol{f_3}\otimes\boldsymbol{g_3}, & \boldsymbol{h_{14}} = \boldsymbol{f_3}\otimes\boldsymbol{g_4}, & \boldsymbol{h_{15}} = \boldsymbol{f_3}\otimes\boldsymbol{g_5}, \\
\boldsymbol{h_{16}} = \boldsymbol{f_4}\otimes\boldsymbol{g_1}, & \boldsymbol{h_{17}} = \boldsymbol{f_4}\otimes\boldsymbol{g_2}, & \boldsymbol{h_{18}} = \boldsymbol{f_4}\otimes\boldsymbol{g_3}, & \boldsymbol{h_{19}} = \boldsymbol{f_4}\otimes\boldsymbol{g_4}, & \boldsymbol{h_{20}} = \boldsymbol{f_4}\otimes\boldsymbol{g_5}, \\
\boldsymbol{h_{21}} = \boldsymbol{f_5}\otimes\boldsymbol{g_1}, & \boldsymbol{h_{22}} = \boldsymbol{f_5}\otimes\boldsymbol{g_2}, & \boldsymbol{h_{23}} = \boldsymbol{f_5}\otimes\boldsymbol{g_3}, & \boldsymbol{h_{24}} = \boldsymbol{f_5}\otimes\boldsymbol{g_4}, & \boldsymbol{h_{25}} = \boldsymbol{f_5}\otimes\boldsymbol{g_5},
\end{array}$$

3. Results of tensor product method

The database consists of 210 crash tests. A model was created based on all cases and then validated. Authors prepared the algorithm that returns following factors:

$$a_1 = 14.226162, a_2 = 1.227114, a_3 = -2.172476, a_4 = -1.614918, a_5 = -0.963204,$$
$$a_6 = 0.222792, a_7 = 1.806460, a_8 = 2.351278, a_9 = 2.623473, a_{10} = 2.815825,$$
$$a_{11} = 0.146215, a_{12} = 0.104217, a_{13} = -0.735955, a_{14} = 0.085348, a_{15} = -1.107606,$$
$$a_{16} = 0.183727, a_{17} = -1.515131, a_{18} = 0.036213, a_{19} = -2.885660, a_{20} = -2.175419,$$
$$a_{21} = 0.330407, a_{22} = 2.687119, a_{23} = 1.541317, a_{24} = 2.447123, a_{25} = 2.000493$$

Figure 1 presents the plot of Legendre polynomials tensor product approximation.

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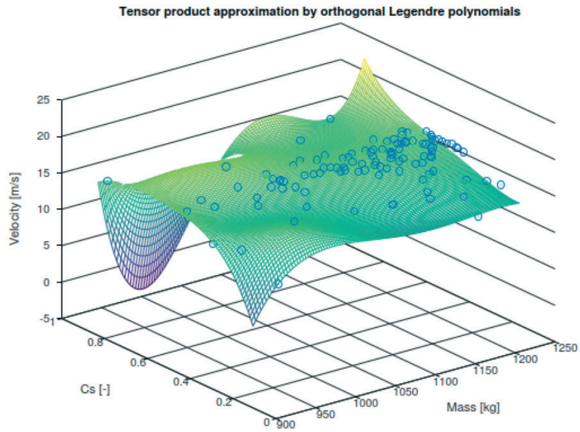

Fig. 1. Tensor product approximation using orthogonal Legendre polynomials

Figure 2 presents the linear approach to approximation of the same database and it shows its inferiority towards the nonlinear approach.

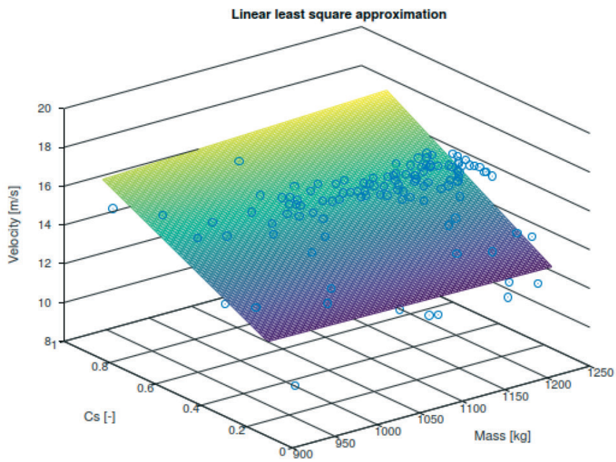

Fig. 2. Least square approximation using linear approach

The relative error for nonlinear approach is **5.7711%**, as shown in Figure 3, whereas, the linear approach reached the value of **6.9180%**, as shown in Figure 4. The difference is not very significant. It is mainly caused by the size of the car. New car availability to absorption of energy during crash is similar to old ones. In other car class like Compact, where the difference is much more significant, error between nonlinear model and linear model is much more visible.

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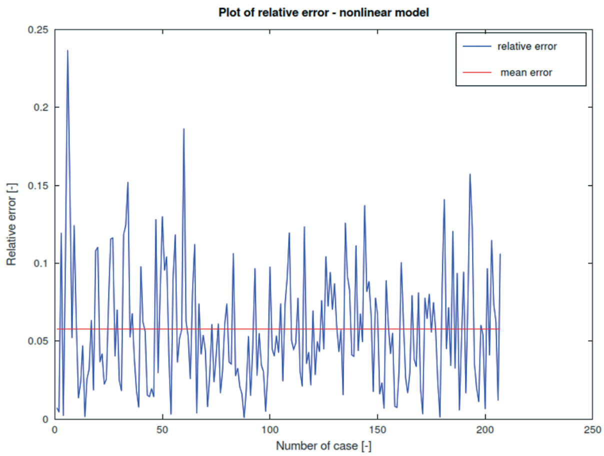

Fig. 3. Value of relative error in nonlinear model

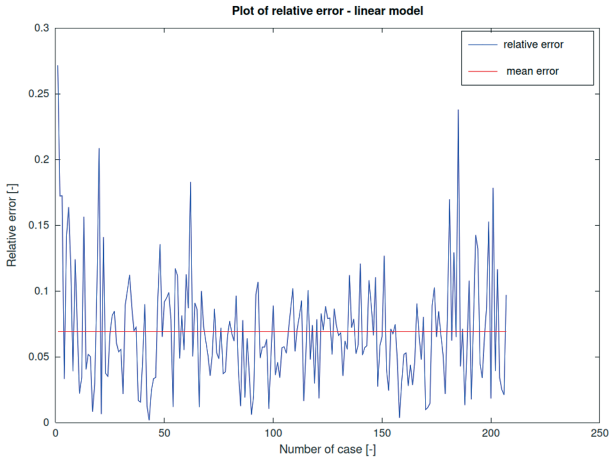

Fig. 4. Value of relative error in linear model

Finally, a comparison of linear and Legendre approach is presented in Figure 5. It's easy to see from this chart that velocity determined by nonlinear model described in this paper is much more accurate than determined by linear ones.

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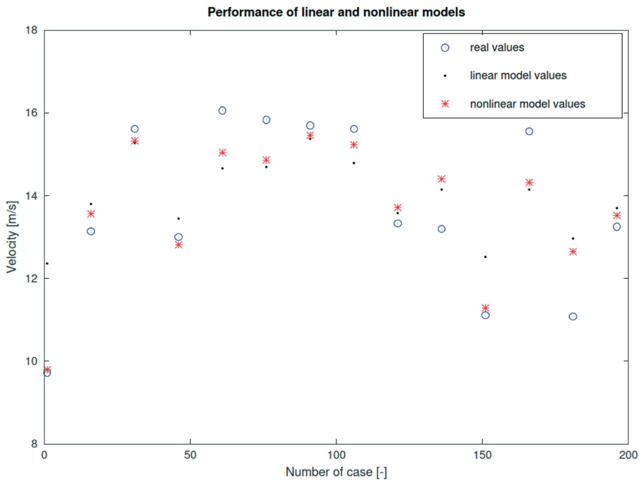

Fig. 5. Performance of linear and nonlinear models (Legendre tensor product)

Table 1 presents detailed data of Legendre approach for representative group.

Table 1. Detailed numerical values of the inverse method

m	C_s	V_t	Expected linear	Expected nonlinear	Linear error	Nonlinear error
948	0.167	9.722	12.362	9.793	0.272	0.007
1098	0.396	13.139	13.798	13.562	0.050	0.032
1202	0.606	15.611	15.272	15.328	0.022	0.018
1090	0.343	13.000	13.446	12.814	0.034	0.014
951	0.540	16.056	14.660	15.044	0.087	0.063
1048	0.536	15.833	14.693	14.862	0.072	0.061
1224	0.618	15.694	15.375	15.458	0.020	0.015
1166	0.539	15.611	14.788	15.231	0.053	0.024
1015	0.364	13.333	13.579	13.712	0.018	0.028
1200	0.444	13.194	14.147	14.404	0.072	0.092
1116	0.204	11.111	12.521	11.290	0.127	0.016
1213	0.443	15.556	14.147	14.319	0.091	0.080
1229	0.274	11.083	12.965	12.647	0.170	0.141
1144	0.380	13.250	13.700	13.522	0.034	0.021

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4. Calculation example

Exemplary calculations were made on the basis of the NHTSA crash test result. The method of measuring frontal deformation is similar to Crash3 method. Photo of frontal deformation is shown in Figure 6.

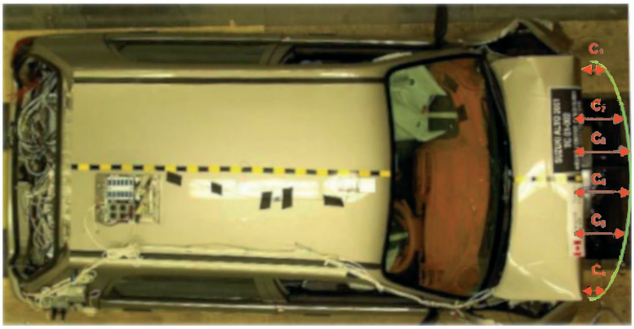

Fig. 6. Frontal deformation with method of measurements

Table 2 represents measurement of deformation. The mass of tested vehicle according to NHTSA report is equal **m = 1057 kg**

Table 2. Measurements of deformation

C_1	C_2	C_3	C_4	C_5	C_6
181 mm	310 mm	322 mm	312 mm	280 mm	151 mm

Average deformation can be calculated using equation below.

$$C_s = \frac{1}{5}\left(\frac{C_1}{2} + C_2 + C_3 + C_4 + C_5 + \frac{C_6}{2}\right) = 278\ mm = 0.278\ m \tag{11}$$

Average deformation and mass of vehicle has to be substituted to equation (10). The result for this case is shown below.

$$EES = 13,6\frac{m}{s} \tag{12}$$

5. Conclusions

The approach of nonlinear approach to precrash vehicle velocity determination, proposed by the Authors, shows promising results. Mean error for Subcompact class is not much better than in linear ones, but the biggest advantage is visible in the Figure 5. Velocity determined using nonlinear method is much more accurate than in linear ones. The difference is not

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as significant as in another class described in other papers done by the Authors but improvement is visible. What is more, authors intend to develop this method by including more factors to decrease the relative error values even further. The superiority of the nonlinear approach is evident, especially when the whole spectrum of examined cases is taken into consideration. After analyzing all classes, authors intend to create a program that will allow to easy apply the described methods in practice.

6. Nomenclature

EES	Equivalent Energy Speed [m/s]
NHTSA	National Highway Traffic Safety Administration
C_s	deformation ratio [m]
C_1–C_6	deformation coefficients
L_t	dent zone width [m]
V_t	vehicle speed [m/s]
W_{def}	work of deformation [J]
b_k	constant slope factor [m/s/m]
m	weight of car [kg]
n	number of cases [–]

7. References

[1] Axler S.J.: Linear Algebra Done Right; 2nd ed., Springer: New York, 1997.

[2] Campbell B.J.: The Traffic Accident Data Project Scale. Proceedings of Collision Investigation Methodology Symposium. 1969.

[3] Cao Y., Luo Y.F.: The Synthesized Method Based on Classical Mechanics and Finite Element for Vehicle Collision Accident Reconstruction Analysis. International Journal of Crashworthiness. 2021, 1–8, DOI:10.1080/13588265.2021.2008741.

[4] Cheney W., Kincaid D.: Numerical Analysis: Mathematics of Scientific Computing. Wadsworth Group, 2002.

[5] Faraj R., Holnicki-Szulc J., Knap L., Seńko J.: Adaptive Inertial Shock-Absorber. Smart Materials and Structures. 2016, 25, DOI:10.1088/0964-1726/25/3/035031.

[6] Geigl B.C., Hoschopf H., Steffan H., Moser A.: Reconstruction of Occupant Kinematics and Kinetics for Real World Accidents. International Journal of Crashworthiness. 2003, 8, 17–27, DOI:10.1533/ijcr.2003.0217.

[7] Gidlewski M., Żardecki D.: Simulation-Based Sensitivity Studies of a Vehicle Motion Model. Transport Means – Proceedings of the International Conference. 2016, 135091, 236–240.

[8] Grolleau V., Galpin B., Penin A., Rio G.: Modelling the Effect of Forming History in Impact Simulations: Evaluation of the Effect of Thickness Change and Strain Hardening Based on Experiments. International Journal of Crashworthiness. 2008, 13, 363–373, DOI:10.1080/13588260801976120.

[9] Han I., Kang H., Park J.C., Ha Y.: Three-Dimensional Crush Measurement Methodologies Using Two-Dimensional Data. Transactions of the Korean Society of Automotive Engineers. 2015, 23, 254–262, DOI: 10.7467/ksae.2015.23.3.254.

https://doi.org/ 10.14669/AM/155005

[10] Han I.: Vehicle Collision Analysis from Estimated Crush Volume for Accident Reconstruction. International Journal of Crashworthiness. 2019, 24(1), 100–105, DOI: 10.1080/13588265.2018.1440499.

[11] Han I.: Analysis of Vehicle Collision Accidents Based on Qualitative Mechanics. Forensic Science International. 2018, 291, 53–61, DOI: 10.1016/j.forsciint.2018.08.004.

[12] Hight P.V., Fugger T.F., Marcosky J.: Automobile Damage Scales and the Effect on Injury Analysis. SAE Technical Paper. 1992, 90092, DOI: 10.4271/920602.

[13] Iraeus J., Lindquist M.: Pulse Shape Analysis and Data Reduction of Real-Life Frontal Crashes with Modern Passenger Cars. International Journal of Crashworthiness. 2015, 20, 535–546, DOI: 10.1080/13588265.2015.1057005.

[14] Krukowski M., Kubiak P., Mrowicki A., Siczek K., Gralewski J.: Non-Linear Method of Determining Vehicle Pre-Crash Speed Based on Tensor B-Spline Products with Probabilistic Weights – Intermediate Car Class. Forensic Science International. 2018, 293, 7–16, DOI: 10.1016/j.forsciint.2018.10.011.

[15] Kubiak P.: Work of Non-Elastic Deformation against the Deformation Ratio of the Subcompact Car Class Using the Variable Correlation Method. Forensic Science International. 2018, 287, 47–53, DOI: 10.1016/j.forsciint.2018.03.033.

[16] Lindquist M., Hall A., Björnstig U.: Real World Car Crash Investigations – A New Approach. International Journal of Crashworthiness. 2003, 8, 375–384, DOI: 10.1533/ijcr.2003.0245.

[17] Mackay G.M., Hill J., Parkin S., Munns J.A.: Restrained Occupants on the Nonstruck Side in Lateral Collisions. Accident Analysis and Prevention. 1993, 25, 147–152, DOI: 10.1016/0001-4575(93)90054-z.

[18] Mannering F.L., Bhat C.R.: Analytic Methods in Accident Research: Methodological Frontier and Future Directions. Analytic Methods in Accident Research. 2014, 1, 1–22, DOI: 10.1016/j.amar.2013.09.001.

[19] Mchenry B.G.: The Algorithms of CRASH. Southeast Coast Collision Conference. 2001.

[20] Mchenry R.R.: Computer Program for Reconstruction of Highway Accidents. SAE Technical Paper. 1973, 90232, DOI: 10.4271/730980.

[21] Nelson W.D.: The History and Evolution of the Collision Deformation Classification SAE J224 MAR80. SAE Technical Paper. 1981, 810213, DOI: 10.4271/810213.

[22] Neptune J.A.: Crush Stiffness Coefficients, Restitution Constants, and a Revision of CRASH3 & Amp; SMAC. SAE Technical Paper. 1998, 90116, DOI: 10.4271/980029.

[23] Norros I., Kuusela P., Innamaa S., Pilli-Sihvola E., Rajamäki R.: The Palm Distribution of Traffic Conditions and Its Application to Accident Risk Assessment. Analytic Methods in Accident Research. 2016, 12, 48–65, DOI: 10.1016/j.amar.2016.10.002.

[24] Prochowski L., Ziubinski M., Gidlewski M.: Experimental and Analytic Determining of Changes in Motor Cars' Positions in Relation to Each Other during a Crash Test Carried out to the FMVSS 214 Procedure. Proceedings of the 2018 XI International Science-Technical Conference Automotive Safety, IEEE. 2018, DOI: 10.1109/AUTOSAFE.2018.8373302.

[25] Prochowski L., Gidlewski M., Ziubiński M., Dziewiecki K.: Kinematics of the Motorcar Body Side Deformation Process during Front-to-Side Vehicle Collision and the Emergence of a Hazard to Car Occupants. Meccanica. 2021, 56, 901–922, DOI:10.1007/s11012-020-01274-3.

[26] Ptak M., Wilhelm J., Klimas O., Reclik G., Garbaciak L.: Numerical Simulation of a Motorcycle to Road Barrier Impact. Lecture Notes in Mechanical Engineering. 2019, 565–573, DOI: 10.1007/978-3-030-04975-1_65.

[27] Sharma D., Stern S., Brophy J., Choi E.: An Overview of NHTSA's Crash Reconstruction Software WinSMASH. Proceedings of the 20th International Technical Conference on Enhanced Safety of Vehicles. 2007, France, Lyon.

[28] Siddall D.E., Day T.D.: Updating the Vehicle Class Categories. SAE Technical Paper. 1996, DOI: 10.4271/960897.

https://doi.org/ 10.14669/AM/155005

[29] Syad B.A., Salmani E., Ez-Zahraouy H., Benyoussef A.: Computational Method of the Stiffness Coefficients A and B in the Case of Frontal Impact from the Results of the Crash Tests. International Journal of Intelligent Transportation Systems Research. 2021, 19, 587–593, DOI: 10.1007/s13177-021-00266-1.

[30] Vangi D., Cialdai C.: Evaluation of Energy Loss in Motorcycle-to-Car Collisions. International Journal of Crashworthiness. 2014, 19, 361–370, DOI: 10.1080/13588265.2014.899072.

[31] Vangi D.: Simplified Method for Evaluating Energy Loss in Vehicle Collisions. Accident Analysis and Prevention. 2009, 41, 633–641, DOI: 10.1016/j.aap.2009.02.012.

[32] Vangi D.: Vehicle Collision Dynamics: Analysis and Reconstruction, Butterworth-Heinemann. Elsevier-Inc. 2020, 157–191, DOI: 10.1016/B978-0-12-812750-6.00005-6.

[33] Vangi D., Cialdai C., Gulino M.S.: Vehicle Stiffness Assessment for Energy Loss Evaluation in Vehicle Impacts. Forensic Science International. 2019, 300, 136–144, DOI: 10.1016/j.forsciint.2019.04.031.

[34] Vangi D., Begani F., Spitzhüttl F., Gulino M.S.: Vehicle Accident Reconstruction by a Reduced Order Impact Model. Forensic Science International. 2019, 298, 426.e1–426.e11, DOI: 10.1016/j.forsciint.2019.02.042.

[35] Wach W.: Reconstruction of Vehicle Kinematics by Transformations of Raw Measurement Data. Proceedings of the 2018 XI International Science-Technical Conference Automotive Safety, IEEE. 2018, DOI: 10.1109/AUTOSAFE.2018.8373324.

[36] Wach W.: Spatial Impulse-Momentum Collision Model in Programs for Simulation of Vehicle Accidents. Proceedings of the 2020 XII International Science-Technical Conference Automotive Safety, IEEE. 2020, DOI: 10.1109/AUTOMOTIVESAFETY47494.2020.9293494.

[37] Wood D.P., Simms C.K.: Car Size and Injury Risk: A Model for Injury Risk in Frontal Collisions. Accident Analysis and Prevention. 2002, 34, 93–99, DOI: 10.1016/s0001-4575(01)00003-3.

[38] Żuchowski A.: The Use of Energy Methods at the Calculation of Vehicle Impact Velocity. The Archives of Automotive Engineering – Archiwum Motoryzacji. 2015, 68(2), 85–111.