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ANALYSIS OF KINEMATICS AND DYNAMICS OF THE VEHICLE DRIVE SYSTEM WITH A PARALLEL POWER FLOW TRANSMISSION

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Summary

A method that makes it possible to analyse a vehicle drive system with a compound parallel power flow transmission even as early as at the project design stage has been presented. The essence of this method lies in determining the values of two characteristic transmission speed ratios i_1 and i_2 of the compound system at which the power transmitted by the continuously variable transmission (CVT) changes its sign, referred to as "zero-power" speed ratios. These speed ratios determine the power curve in the whole speed ratio regulation range from i_1 to i_2 and thus, the speed and torque curves at the CVT input and output shafts. Thanks to this, the structure of the compound transmission can be so selected that the loads of individual CVT subassemblies would not exceed the acceptable levels and they would be kept within the predefined limits.

As an example, the drive trains of four gearboxes with parallel power flow, possible to be found at present in Poland, were analysed with the use of this method, i.e. the Voith DIWA and Allison EV Drive models and a prototype built at the Lodz University of Technology (TUL), designed for city buses, and the Toyota Prius or Lexus passenger car gearboxes.

Moreover, a CVT friction gear and a conventional gearbox were analysed for their properties to be compared with those of the parallel power flow systems.

Keywords: automotive transmission, urban traffic, parallel power flow transmission

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1. Introduction

The more and more stringent limits gradually imposed on motor vehicle exhaust emissions by new legal regulations have resulted in considerable intensification of works on new technological solutions, which include hybrid drive systems. In consequence, an increasingly dynamic process of providing motor vehicles with such systems can be observed, in spite of growing costs of production of vehicles of this type. The favourable effects of this process include lower consumption of petroleum-derivative fuels as well as reduced exhaust emissions. By 2020, about 50 new vehicle models with hybrid drive systems are to be launched, according to Bosch forecasts [2].

The fact that the internal combustion (IC) engine was chosen as the primary mover of motor vehicles has resulted in the introduction of such components as clutch, gearbox, and final drive with a differential gear to the vehicle drive system. This is because the power curve of the IC engine (inverted parabola) completely differs from the hyperbolic shape, which is considered ideal. For the over 100-year motor vehicle history, the main challenge to automobile engineers was the achieving of the best possible vehicle dynamics. The present-day exhaust emission regulations force corrections in the construction of vehicle drive systems. The least efficient component of the powertrain is the IC engine, whose average efficiency in the urban traffic is about 5-9 % as against the level of 35-40% achievable in steady-state conditions. In the extra-urban traffic, the vehicle engine can be "forced" to operate within the area of its maximum efficiency by the introduction of economical gear ratios and the following of an appropriate (economical) way of driving. This is connected with the constant speed ratio between vehicle wheels and its engine in the conventional drive system, where the area of engine operation parameters is determined by the actual vehicle motion.

At the highly dynamic "urban" way of driving, the vehicle's IC engine predominantly operates in the areas characterized by low efficiency and high exhaust emissions; conversely, when the vehicle is driven in a way such as on a motorway, i.e. smoothly and with long-lasting periods of stable speed, its engine operates in the areas of the highest possible efficiency and minimum exhaust emissions. Hence, if an IC engine is to be the prime mover of a vehicle, a solution should be sought where in the "urban" traffic conditions, the vehicle speed would be controlled by changing the transmission ratio with the engine operating with stable working parameters that would ensure its maximum efficiency while in the "motorway" traffic conditions, the vehicle would function as if it were provided with a conventional gearbox used previously because of the maximum efficiency achieved then by all vehicle components. These objectives may be achieved with the use of transmissions with parallel power flow. Additionally, if an appropriate type of the continuously variable transmission were chosen, it would become possible to control the power flow in both directions, i.e. to and from the vehicle wheels, thanks to which the braking energy might be recuperated and then stored and utilized for accelerating the vehicle provided with a hybrid drive system ("regenerative braking") [3, 4, 5].

An analysis of various solutions of the hybrid drive system, carried out within the work described in [3], has revealed the following good points of this concept:

- possibility of reducing the engine operation area to a line segment or even a point, depending on the kinematic structure adopted;
- high average engine operation efficiency and, in consequence, a reduction in the operating fuel consumption in the "urban" traffic conditions to a level observed in the "motorway" traffic conditions, in spite of the power transmission efficiency being reduced by several percent in comparison with that of the conventional power transmission system;
- smooth change in the speed and direction of vehicle motion (i.e. change from the forward motion to reversing and vice versa);
- driving the vehicle with as low a speed as required, inclusive of the cases of sudden changes in the resistance to motion, e.g. on roads with varying slopes;
- small dimensions and mass in relation to the power transmitted;
- possibility of braking the vehicle "with any intensity" even to a stop, with the maximum deceleration depending on the power capacity of the continuously variable transmission (CVT).

The obtaining of all the above benefits depends on the properties of the specific CVT used. Among the existing continuously variable transmissions, the tasks set to them are most fully performed by hydrostatic transmissions and then, in descending order, electrical transmissions, hydrokinetic torque converters, and friction gears.

2. Examples of transmission systems with parallel power flow

In Poland, four designs of gearboxes with parallel power flow can now be met in service, i.e. the Voith DIWA and Allison EV Drive models and a prototype built at the Lodz University of Technology (TUL), designed for city buses, and the Toyota Prius or Lexus passenger car gearboxes.

In the Voith design, a hydrokinetic torque converter has been used as the CVT unit; in the Allison and Toyota designs, electrical transmissions have been applied because of the concept of a hybrid drive system with electric storage batteries having been adopted. In the prototype built at the Lodz University of Technology (hereinafter referred to as "TUL prototype"), a hydrostatic transmission has been used in consideration of the unit power capacity of the energy-transmitting devices available in Poland in early 1990s (e.g. hydro-pneumatic accumulators).

The transmissions mentioned above may be used as examples for the assessment of usability and quality of the parallel power flow transmission as an environment-friendly solution.

2.1. The Voith DIWA transmission

The history of parallel power flow transmissions dates back to 1940s, when the Voith Diwabus design appeared. The DIWA transmissions manufactured by Voith at present have

been developed from that old concept with taking into account the experience accumulated since 1950. They are fully automatic transmissions, making it possible to brake the vehicle with the use of a retarder and thus meeting the European standards, which require that a third separate permanent brake should be provided in vehicles of specific categories. Alas, the energy of braking the vehicle (actually, a bus) is lost in this case.

The main component of the transmission is a hydrokinetic torque converter, see Fig. 1 [9]. Viewed from the engine end, it is preceded by converter pump brake 2 and clutches 3 and 5 of gearset 4, which splits the IC engine output power. Behind the hydrokinetic torque converter, there is planetary gearset 6, in which the power parts flowing in the mechanical and hydraulic branches of the system are summed up, and planetary gearset 7, which makes it possible to switch the transmission to reverse and to use the transmission as a retarder. In neutral, all the transmission clutches and brakes are disengaged.

The hydrokinetic torque converter is used to accelerate the vehicle in the 1st gear and as a hydrodynamic retarder when braking. In the 1st gear, the engine power is transmitted in the system with parallel power flow, in which the hydrokinetic torque converter is connected in parallel with the mechanical transmission. In the 1st gear, clutch 5 and the brake of summing-up gearset 6 are engaged. Clutch 5 connects the transmission input shaft with the ring gear of splitting gearset 4 and the brake immobilizes the ring gear of gearset 6. The engine power is transmitted to the output shaft in parallel, i.e. partly via a hydraulic branch and, in another part, via a mechanical branch (see Fig. 1), and the power is split in a way as described below.

- When the vehicle is at a standstill, the transmission output shaft and the planet carrier of splitting gearset 4 do not move. The pump impeller rotates with a speed many times as high as that of the engine output shaft. The torque converter and the speed ratio of the splitting gearset are so selected that when the engine has been started up and the converter has been completely filled with transmission fluid, the engine is loaded to a level of 60 %, thanks to which it operates in the range of the maximum output torque and minimum fuel consumption. The turbine torque is transmitted to the output shaft via summing-up gearset 6.
- With rising vehicle speed, i.e. with growing angular speed of the transmission output shaft, the power part transmitted hydraulically by the torque converter decreases while the power part transmitted mechanically by the sun gear in gearset 4 increases. At the same time, the IC engine speed increases and the speeds of the converter pump impeller and the sun gear of splitting gearset 4 decline to reach zero at a specific vehicle speed; at this instant, brake 2 gets engaged and the brake of summing-up gearset 6, coupled with the turbine rotor, gets released. From then, the engine power is transmitted in a purely mechanical way to the transmission output shaft, via clutch 5 and splitting gearset 4 in the 2nd gear or via clutch 3 and splitting gearset 4 in the 3rd gear, i.e. similarly as it is in the conventional design.

In the 1st gear, active in a very wide speed range, the vehicle may be accelerated to $0.4 V_{\max}$. The splitting of power and the functioning of the torque converter in a system with controllable internal speed ratio, with no shifting of gears in this phase of vehicle drive, translates into greater safety of vehicle operation, higher ride comfort, and longer

service life of vehicle components. Thanks to the splitting of power, both the good points of hydrodynamic systems, such as high value of the driving force, smooth acceleration, and automatic adaptation to current load conditions, and the advantages of transmissions with parallel power flow, i.e. high efficiency and possibility of forcing the engine to function in the area of economical and environment-friendly operation, can be fully utilized.

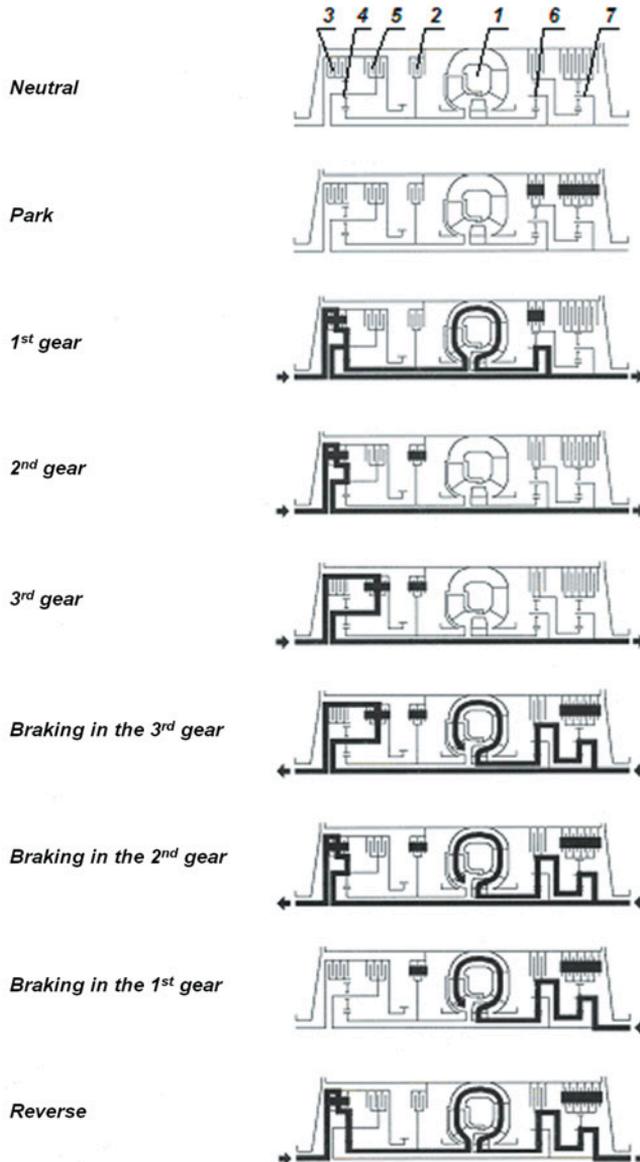


Fig. 1. Power flow paths in individual gears [9]

The reversing is done with the use of splitting gearset 4, hydrokinetic torque converter 1, and reversing planetary gearset 7. The achievable driving force is equal to that developed in the 1st gear and the maximum driving speed is equal to $0.1 V_{\max}$ (0.1 of the maximum speed of the vehicle when driven forwards).

In the latest design version of this transmission, four gears have already been applied, thanks to which the IC engine operation range may be considerably narrowed and thus the average efficiency of the engine may be raised because the hydrokinetic torque converter is used in the 1st gear only, as shown in Fig. 1. In the other gears, the engine cooperates with the transmission as in conventional systems and the engine operation range should be reduced to the maximum engine efficiency area.

The trend to increase the number of gears can also be noticed in automatic transmissions developed by other manufacturers.

2.2. The GM Allison EV Drive transmission

In the two-mode Allison E^V Drive transmission (Fig. 2) [7, 10], the major parts are: dual-ratio planetary gearset 3 coupled by clutch 2 with engine 1, planetary gearset 4 with two clutches 5 and 6 to change the transmission operation modes, and two electric machines 7 and 8 functioning alternately as motors or generators (referred to as motor/generator units). The essence of such a concept lies in the fact that the function of controlling the vehicle speed has been shifted from the engine to the transmission with parallel power flow, thanks to which the engine may operate with a constant speed of its output shaft regardless of the vehicle speed.

Additionally, the design parameters of the planetary gearsets have been so selected that for every transmission operation mode, there is a specific vehicle speed at which the planetary gearsets behave as if they were blocked into rigid units, i.e. all the gearset components rotate with an identical angular speed. At such speeds, the transmission operates with its maximum efficiency and the corresponding values of the vehicle speed (bus speed in this case) are 22 km/h and 87 km/h (points "A" and "B", respectively, in the graph shown in Fig. 3 [10]). The rotational speeds of the motor/generator units and of the vehicle engine are determined by the vehicle speed and kinematics of both planetary gearsets.

In transmission operation mode I, clutch 5 is engaged, clutch 6 is disengaged, and planetary gearset 4 operates as a reduction gear. Motor/generator unit 8 operates as a motor and drives the sun gear of gearset 4 and the sun gear of the second stage of gearset 3. Motor/generator unit 7 operates as a motor at lower speeds and as a generator at the top end of the speed range and this is a consequence of the energy losses that begin to take place in the transmission with parallel power flow when the transmission goes through the "zero-power" working point. The vehicle reversing is implemented by forcing the motor/generator unit 8 to rotate in the opposite direction.

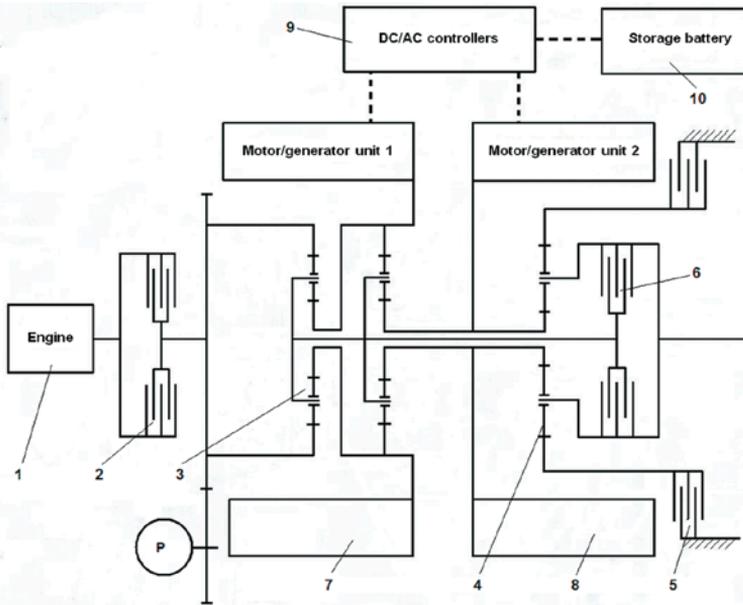


Fig. 2. Schematic diagram of the Allison EV Drive hybrid power transmission system [7, 10]:
 1 – IC engine; 2 – clutch; 3 – dual-ratio planetary gearset; 4 – planetary gearset;
 5 – mode I clutch; 6 – mode II clutch; 7 and 8 – motor/generator units; 9 – voltage inverters;
 10 – NiMH battery pack

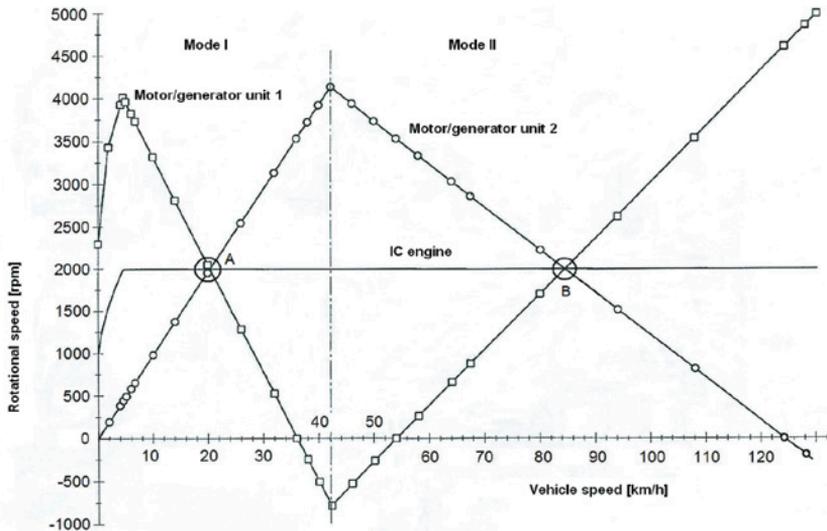


Fig. 3. Graph representing the rotational speeds of major components of the Allison EV Drive hybrid power transmission system [10]

In transmission operation mode II, clutch 6 having been engaged transmits power from the dual-ratio planetary gearset 3 directly to the transmission output shaft. Clutch 5 remains then disengaged and motor/generator unit 7, operating as a motor, drives the gearset 3. Motor/generator unit 8 operates then as a generator.

The electric machines generate and use alternating current (AC) and the storage battery pack operates with direct current (DC). Therefore, the dual power inverter module converts direct current into alternating current or vice versa, depending on the need, and thus facilitates energy storage and transmission. The module operates in the voltage range 430-900 V and it is controlled with the use of a CAN communication system.

The energy storage system is based on nickel-metal hydride NiMH cells, which offer one of the highest energy density values (360 MJ/m^3). The energy stored is generated during normal operation of the electric machines and when the vehicle is braked. The energy storage system of 600 V nominal voltage is cooled by air.

Operational examinations carried out in the USA on hybrid buses with Cummins diesel engines of 209 kW rated power, provided with Allison E^v Drive power transmissions having two 75 kW electric machines each, revealed that the average fuel consumption in the Manhattan, OCTA, and CBD urban test cycles decreased by 51 %, the NO_x , CO, H_nC_m , and CO_2 emissions were reduced by 28 %, 29 %, 43 %, and 36 %, respectively, and the particulate matter emissions dropped even by as much as 66 %, in comparison with the corresponding test results obtained for buses with conventional drive systems [7].

At the beginning of 2007, the drive system to the Allison design was introduced for the first time in Europe to the series production of city buses by Solaris Bus & Coach, a Polish bus manufacturer.

Buses with such drive systems can be seen in the streets of many European cities, e.g. Bremen, Bochum, Dresden, Hannover, Leipzig, Munich, Lenzburg, and Poznań. In the case of the Solaris hybrid bus, the fuel consumption was found to have decreased by 20-25 %, and the emissions of the pollutants mentioned above were reduced as follows: NO_x by 39 %, CO by 10 %, CO_2 by 23 %, H_nC_m by 14 %, and particulate matter by 6 % [2].

2.3. The TUL prototype transmission

The TUL prototype gearbox (Fig. 4) consists of a planetary gear unit with three gearsets I, II, and III, clutches "A" and "B" used to change transmission ratios, hydrostatic transmission "a" and "b", and a few gear pairs with fixed axes. The speed ratio of the three-range transmission is altered by changing the speed ratio of the hydrostatic transmission. The following combinations are possible: in range I, the hydrostatic transmission is connected with the planetary gear unit by means on a fixed-axes gear train, while in ranges II and III, internal connections are utilized [1, 2]. The method of connecting the hydrostatic branch with the mechanical one has an effect on the shape of the curves representing the basic transmission parameters, Fig. 4, and on the method of controlling the transmission system as a whole.

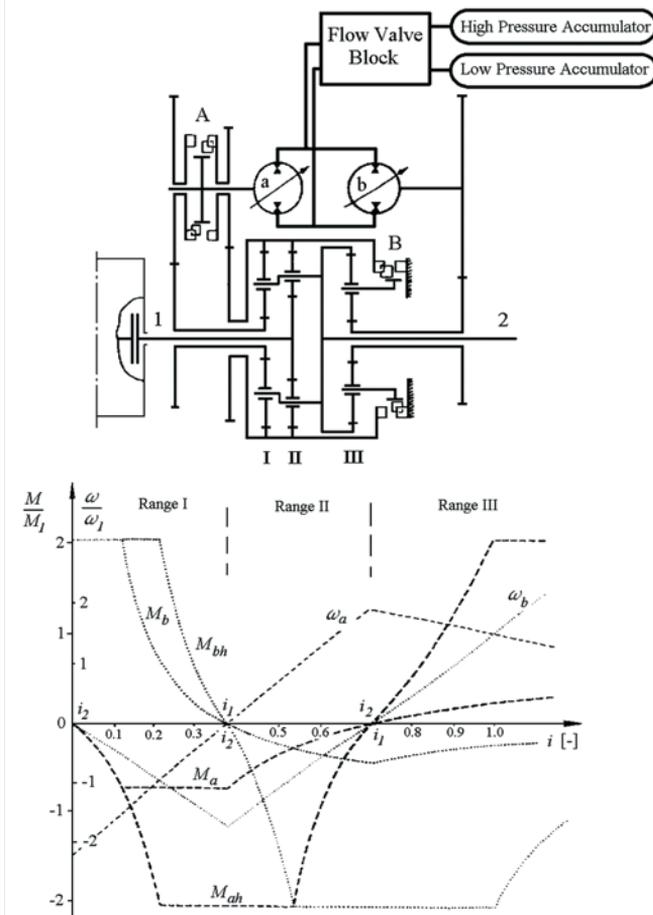


Fig. 4. Schematic diagram of the TUL prototype drive system and curves representing the basic parameters of the hydrostatic transmission

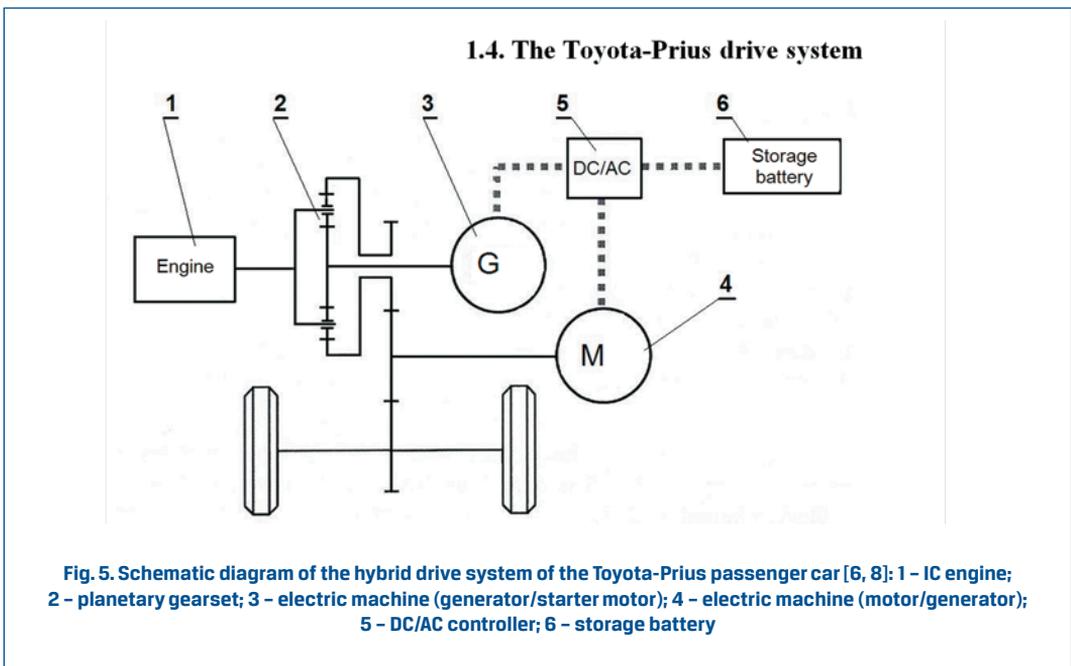
In neutral, transmission output shaft does not rotate, clutches "A" and "B" are shifted to the right, pump "a" is set to zero delivery and motor "b" is set to maximum displacement. Pump "a" is driven by the IC engine through the sun gear in gearset I and gear wheels with fixed axes. The shaft of motor "b" does not move. In result of an inclination of the swashplate of machine "a", machine "b" is driven with an angular speed of ω_b corresponding to the swashplate inclination angle and this causes transmission output shaft 2 to rotate as well. The speed ratio $i = \omega_2/\omega_1$ is initially raised by an increase in the delivery of machine "a" to a maximum and then by a reduction in the displacement of machine "b", which operates in this range as a hydrostatic motor, even to zero. Simultaneously, the angular speed of

machine "a" gradually declines to zero, too. The angular speeds of ring gears 2 and 4 and of the other side of clutch "B" are reduced proportionally, too.

When machine "a" stops ($\omega_a = 0$), the transmission operation range changes from I to II and clutch "B" connects gearset III with ring gears 2 and 4. At this instant, machine "b" begins to operate as a pump and machine "a" takes over the role of a motor in the hydrostatic system, whose total speed ratio is further changed by raising the delivery of machine "b" from zero to a maximum and reducing the displacement of machine "a".

Transmission operation range II ends when machine "b" stops ($\omega_b = 0$). At this moment, clutch A is shifted to its left position, the system goes into range III, and the speed ratio is changed identically as it is done in range I.

The limits of the angular speeds of machines "a" and "b" are determined by the design of these machines and by the maximum value of fluid pressure in the hydrostatic system. In Fig. 1, symbols M_{ah} and M_{bh} denote the curves representing the torques on shafts of machines "a" and "b", respectively, obtained as sums of the torques coming from two sources, i.e. from the IC engine and from the hydro-pneumatic accumulators. For the system being exclusively driven by the IC engine, these torques are denoted by M_a and M_b , respectively. The evident growth in the torque values must result in increased dynamics of the bus.



In the hybrid drive system of the Toyota-Prius passenger car, the vehicle may be driven either by its electric motor only (at low acceleration or when moving with a constant speed) or both by its electric motor and IC engine (at higher loads), see Fig. 5 [6, 8]. The IC engine is

also engaged when the battery is in a low state of charge. The amount of the electric power used to drive the vehicle is regulated by controller 5, which monitors the state of charge of battery 6 and, based on this, sets the appropriate proportion between the electric power parts drawn from the battery and from the IC engine to obtain the maximum total efficiency of the drive system in specific load conditions.

When the vehicle is braked, the IC engine is switched off and controller 5 switches electric machine 4 to the generating mode, thanks to which the kinetic energy of the vehicle can be recovered and stored in battery 6.

The rotational speeds of electric machines 3 and 4 and of IC engine 1 are determined by the kinematics of the planetary gearset and the configuration of connections between individual component units of the drive system. The curves representing the speeds have been shown in Fig. 6. Like in the design of the Allison E^V Drive transmission, Toyota engineers also distinguished the vehicle speed of about 50 km/h, at which all the parts of the planetary gearset rotate with an identical angular speed. In such conditions, the gearset behaves as if it were blocked into a rigid unit and, in consequence, the power is transmitted with maximum efficiency.

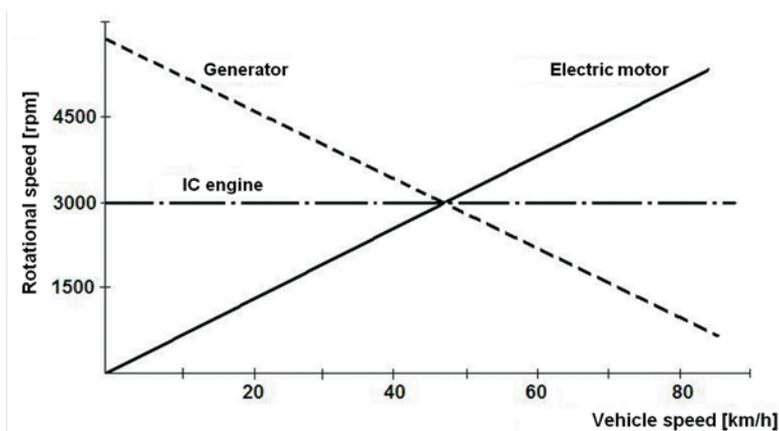


Fig. 6. Graphical representation of the rotational speeds of major components of the hybrid drive system of the Toyota-Prius passenger car [8]

The electric machines used are high-efficiency brushless permanent magnet synchronous machines of 50 kW power capacity at 500 V rated voltage and of 400 Nm maximum torque, which is practically constant over the 0 to 1540 rpm speed range. The total power available from both the power sources reaches 85 kW at vehicle speeds exceeding 80 km/h.

The system includes an IC engine of 1.5 dm³ capacity, which can deliver a peak power output of 57 kW at 5000 rpm and a maximum torque of 115 Nm at 4200 rpm.

The manufacturer claims [4] that the fuel consumption remains on a level of 3.5 dm³/100 km and the exhaust emissions have been reduced by 50 % in comparison with those of conventionally powered cars.

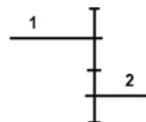
In 2009, the 3rd generation of this drive system was launched; the system being manufactured at present has been developed as its 4th generation, provided with a 72 kW IC engine and a 59 kW electric motor.

The presented solutions of the parallel power flow drives, put into series production, have been confirmed as being suitable for urban traffic. Their common feature is splitting the power delivered by the IC engine into two power flows with the use of a planetary gear unit and adding the power flows to each other with the use of a constant-ratio gear train. As it will be shown in a subsequent part of this article, this is the only option of the system architecture that makes it possible to control vehicle speed from zero up with the speed ratio being also changed from zero up in a continuously variable transmission (CVT).

3. Kinematic and dynamic analysis of parallel power flow transmissions

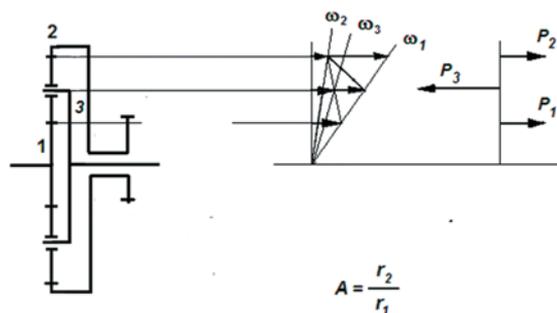
The properties of a parallel power flow transmission can be defined if the kinematic parameters of individual gearsets incorporated in the transmission are known; this may be done with the use of a method as presented below [1, 3].

The properties of a gearset with fixed axes are described by its speed ratio and efficiency, i.e.:



$$\begin{aligned} \omega_1 - i_{12}\omega_2 &= 0 \\ M_1 : M_2 &= 1 : -i_{12} \eta_{12} \end{aligned} \quad (1)$$

For a planetary gearset, we have:



$$\begin{aligned} \omega_1 + A\omega_2 - (1+A)\omega_3 &= 0 \\ M_1 : M_2 : M_3 &= 1 : A\eta_{12} : -(1+A)\eta_{13} \end{aligned} \quad (2)$$

The α and β members (subassemblies) of the hydrostatic transmission may operate interchangeably as a pump and motor, in both directions of the power flow.

In result of solving the kinematic equations, the following relations between the angular speeds of individual system elements are obtained:

$$\frac{\omega_1}{D_1} = \frac{\omega_2}{D_2} = \frac{\omega_3}{D_3} = \dots = \frac{\omega_{p-1}}{D_{p-1}} = \frac{\omega_p}{D_p} \tag{5}$$

where D_1, D_2, \dots, D_p are determinants obtained from the matrix

$$\begin{vmatrix} a_{11} & a_{12} & a_{1a} & a_{1b} & a_{1c} & a_{1d} & a_{1p} \\ a_{21} & a_{22} & a_{2a} & a_{2b} & a_{2c} & a_{2d} & a_{2p} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ a_{(p-2)1} & a_{(p-2)2} & a_{(p-2)a} & a_{(p-2)b} & a_{(p-2)c} & a_{(p-2)d} & a_{(p-2)p} \\ 0 & 0 & i_z & -i_z & -1 & 1 & 0 \end{vmatrix} \tag{6}$$

by omitting the appropriate column in each case.

The total kinematic ratio of the transmission as a whole is:

$$i = \frac{\omega_2}{\omega_1} = \frac{D_2}{D_1} \tag{7}$$

or, after solving the determinants:

$$i = \frac{i_z (D_2^a - D_2^b) + (D_2^c - D_2^d)}{i_z (D_1^a - D_1^b) + (D_1^c - D_1^d)} = \frac{A i_z + B}{C i_z + D} \tag{8}$$

where:

D_n^w – minor determinants obtained from the matrix by omitting the last row and columns n and w , corresponding to the elements n and w for which the speed ratio is to be defined;

A, B, C, D – constant quantities obtained by transformations in accordance with the formula above.

The value of the speed ratio of the CVT may be determined from the above equation as a function of the total ratio of the parallel power flow transmission (often referred to as "parallel transmission"), in the form as follows:

$$i_z = \frac{D i - B}{A - C i} \tag{9}$$

The value of the power N_q flowing through branch q of a compound transmission should be determined from the Krejnes equation [1, 3]:

$$\frac{N_q}{N} = \frac{i_q}{i} \frac{\delta i}{\delta i_q} \tag{10}$$

where:

N – power transmitted by a compound parallel transmission;

N_q – power transmitted by branch q of the transmission;

i – total ratio of the compound parallel transmission;

i_q – speed ratio of branch q of the transmission.

If the transmission branch includes a CVT, the following is obtained:

$$\frac{N_z}{N} = \frac{i_z}{i} \frac{\delta i}{\delta i_z} = \frac{i_z (AD - BC)}{i (C i_z + D)^2} \quad (11)$$

and if the appropriate expression is substituted for i_z then the above will take the form:

$$\frac{N_z}{N} = \frac{(Di - B)(A - Ci)}{i(AD - BC)} \quad (12)$$

Having introduced the following denotation:

$$\frac{A}{C} = i_1 \text{ and } \frac{B}{D} = i_2 \quad (13)$$

we obtain:

$$\frac{N_z}{N} = \frac{(i_1 - i)(i - i_2)}{i(i_1 - i_2)} \quad (14)$$

The angular speeds at the inputs and outputs of individual system members are directly defined by solutions of the set of kinematic equations; as an example, the following holds for the CVT:

$$\frac{\omega_\alpha}{\omega_1} = \frac{\omega_a - \omega_b}{\omega_1} = \frac{D_a - D_b}{D_1} = k_\alpha \frac{i_1 - i}{i_1 - i_2} \quad (15)$$

$$\frac{\omega_\beta}{\omega_1} = \frac{\omega_c - \omega_d}{\omega_1} = \frac{D_c - D_d}{D_1} = k_\beta \frac{i - i_2}{i_1 - i_2} \quad (16)$$

where:

k_α and k_β – constants independent of parameters i_1 and i_2 and being combinations of minor determinants of the matrix.

The torque equations stem from the above relations and have the form as below:

$$\frac{M_\alpha}{M_1} = \frac{1}{k_\alpha} \frac{i - i_2}{i} \quad (17)$$

$$\frac{M_\beta}{M_1} = \frac{1}{k_\beta} \frac{i_1 - i}{i} \quad (18)$$

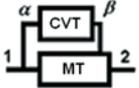
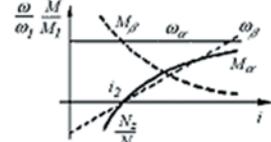
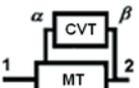
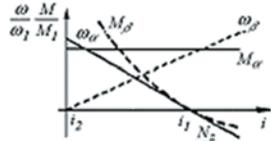
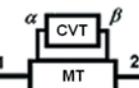
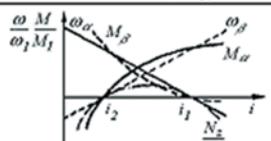
The equations shown above make it possible to calculate the power, torque, and angular speed values for all the branches of a compound parallel transmission system at an assumption of ideal efficiency.

3.1. Classification of parallel transmissions

Thanks to the introduction of two characteristic speed ratios i_1 and i_2 of the compound transmission system at which the power transmitted by the CVT goes through zero, a simple classification of parallel transmissions as presented in Table 1 [1, 3] has become possible. For each of the groups, there are basic relations defining the operation parameters of the internal CVT, independent of the degree of complication of the transmission construction and, instead, exclusively depending on the values of the said two speed ratios i_1 and i_2 . There are three types of transmission design:

- with the CVT being connected with the mechanical transmission branch through a constant-ratio gear train situated at the system input;
- with the CVT being connected with the mechanical transmission branch through a constant-ratio gear train situated at the system output;
- with an internal connection, i.e. with the CVT being connected with the mechanical transmission branch through planetary gearsets situated at the system input and output.

Table 1. Classification of the gearboxes with the parallel power flow

	Structure, schematic	Load characteristics of the CVT	CVT operation parameters	
Connection at the input			$0 < i_2 = \frac{B}{D} < i_1 = \infty$ $\frac{\omega_{\alpha}}{\omega_1} = k_{\alpha}$ $\frac{M_{\alpha}}{M_1} = \frac{1}{k_{\alpha}} (1 - \frac{i_2}{i})$	$\frac{N_2}{N} = 1 - \frac{i_2}{i}$ $\frac{\omega_{\beta}}{\omega_1} = k_{\beta} (i - i_2)$ $\frac{M_{\beta}}{M_1} = \frac{1}{k_{\beta}} \frac{i}{i - i_2}$
Connection at the output			$0 = i_2 < i_1 = \frac{A}{C} < \infty$ $\frac{\omega_{\alpha}}{\omega_1} = k_{\alpha} (1 - \frac{i_1}{i})$ $\frac{M_{\alpha}}{M_1} = \frac{1}{k_{\alpha}}$	$\frac{N_2}{N} = 1 - \frac{i_1}{i}$ $\frac{\omega_{\beta}}{\omega_1} = k_{\beta} \frac{i}{i_1}$ $\frac{M_{\beta}}{M_1} = \frac{1}{k_{\beta}} (\frac{i_1}{i} - 1)$
Internal connection			$0 < i_2 = \frac{B}{D} < i_1 = \frac{A}{C} < \infty$ $\frac{\omega_{\alpha}}{\omega_1} = k_{\alpha} \frac{i_1 - i}{i_1 - i_2}$ $\frac{M_{\alpha}}{M_1} = \frac{1}{k_{\alpha}} \frac{i - i_2}{i}$	$\frac{N_2}{N} = \frac{(i_1 - i)(i - i_2)}{i(i_1 - i_2)}$ $\frac{\omega_{\beta}}{\omega_1} = k_{\beta} \frac{i - i_2}{i_1 - i_2}$ $\frac{M_{\beta}}{M_1} = \frac{1}{k_{\beta}} \frac{i_1 - i}{i}$

Legend: CVT – continuously variable transmission; MT – mechanical transmission

The system with the connection at the input is characterized by the following: the hyperbola of power N_z/N becomes equilateral and angular speed ω_α is constant, proportional to the angular speed of the transmission input shaft (determined by the gear train ratio). In this case, only one zero-power point (i_2) may be utilized and the favourable range of system operation is situated above this point, i.e. at higher values of the transmission ratio. At lower values of this ratio, the phenomenon of power circulation takes place in the transmission.

In the system with the connection at the output, the hyperbola of power N_z/N "degenerates" into its asymptote, in comparison with the system with an internal connection. The same applies to the curve representing torque M_α . A favourable feature is the fact that the value of power N_z/N is limited even at the lowest transmission ratios (when $i \approx 0$). Such a connection, as the only one among the three possible options, makes it possible for the transmission to perform the driving function – from $i_2 = 0$, which corresponds a vehicle speed of $V = 0$.

In the system with an internal connection, there is no proportionality between the system operation parameters occurring in the parallel branches, because both at the input and output, the branches are connected with each other through planetary gearsets, i.e. mechanisms with more than one degree of freedom. The curve representing power N_z/N is a hyperbola, whose asymptotes are the ordinate axis and a sloped line. Between points i_1 and i_2 , the value of power N_z/N reaches its maximum at a transmission ratio of $i = (i_1 * i_2)^{0.5}$:

$$\left| \frac{N_z}{N} \right|_{max} = \frac{\left(\frac{i_1}{i_2} \right)^{0.5} - 1}{\left(\frac{i_1}{i_2} \right)^{0.5} + 1} \quad (19)$$

In a compound transmission with parallel power flow, the zero-point transmission ratios i_1 and i_2 define the curves representing the basic parameters at the input (α) and output (β) of the CVT. Thus, the structure of the compound transmission can be so selected that the loads of individual CVT subassemblies would not exceed the acceptable levels and they would be kept within the predefined limits. If the system is driven from a single prime mover, an additional limitation is imposed by the power capacity of the prime mover. Therefore, with the above findings being taken into account, the maximum vehicle dynamics can be determined even as early as at the project design stage.

3.2. Analysis of selected parallel power flow transmissions

Based on the above relations and on an assumption made that $\eta = 1$ for all the component gearsets, the following can be obtained:

- For the DIWA transmission, the range of transmission ratios in the 1st gear is:

$$i = \frac{A_1 i_z}{(1 + A_1) i_z + (1 + A_2)} \quad (20)$$

Hence:

$A = A_1, B = 0, C = 1 + A_1, D = 1 + A_2, i_2 = 0, i_1 = \frac{A_1}{1 + A_1}$, and $i < 0$ to 0.4 for $i_z < 0$ to $1 >$;

A_1 – parameter of the planetary gearset that splits the power into parallel flows, e.g. $A_1 = 3$;

A_2 – parameter of the planetary gearset that adds the power flows to each other, e.g. $A_2 = 2.5$.

- For the Allison E^v Drive transmission in mode I, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{A_1 (1 + A_2) i_z}{(1 + A_1) (1 + A_3) i_z + (A_1 A_2 - 1) (1 + A_3)} \quad (21)$$

Hence:

$A = A_1 (1 + A_2), B = 0, C = (1 + A_1) (1 + A_3), D = (A_1 A_2 - 1) (1 + A_3)$,

and $i_2 = 0, i_1 = \frac{A_1 (1 + A_2)}{(1 + A_1) (1 + A_3)} \Rightarrow$ when $A_2 = A_3$, then $i_1 = \frac{A_1}{(1 + A_1)}$; (22)

A_1, A_2, A_3 – parameters of successive planetary gearsets.

When $i_z = 1$ (i.e. the angular speeds of both motor/generator units are equal to each other, which corresponds to point A in Fig. 3), then

$$i = \frac{A_1 (1 + A_2) i_z}{(1 + A_1) (1 + A_3) i_z + (A_1 A_2 - 1) (1 + A_3)} = \frac{1 + A_2}{1 + A_3 - A_2 + A_2 A_3}; \quad (23)$$

for $A_2 = A_3 = 3$, we have $i = 0.4$, which corresponds to a transmission ratio value of 2.5 in the conventional system.

- For the transmission in mode II, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{A_1 A_2 i_z + A_1}{(A_1 A_2 - 1) i_z + (1 + A_1)}. \quad (24)$$

Hence:

$A = A_1 A_2, B = A_1, C = A_1 A_2 - 1, D = 1 + A_1, i_2 = \frac{A_1}{1 + A_1}, i_1 = \frac{A_1 A_2}{A_1 A_2 - 1}$. (25)

When $i_z = 1$ (i.e. the angular speeds of both motor/generator units are equal to each other, which corresponds to point B in Fig. 3), then

$$i = \frac{A_1 A_2 i_z + A_1}{(A_1 A_2 - 1) i_z + (1 + A_1)} = 1 \text{ (direct speed gear)}. \quad (26)$$

- For the TUL prototype transmission in range I of speed ratios, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{\frac{z_{12} \cdot z_9}{z_{11} \cdot z_{10}} i_z}{(1 + A_2) \frac{z_{12} \cdot z_9}{z_{11} \cdot z_{10}} i_z + A_1 A_2} . \quad (27)$$

Hence:

$$A = \frac{z_{12} \cdot z_9}{z_{11} \cdot z_{10}} i_z, B = 0, C = (1 + A_2) \frac{z_{12} \cdot z_9}{z_{11} \cdot z_{10}}, D = A_1 A_2, i_2 = 0, i_1 = \frac{1}{1 + A_2} ; \quad (28)$$

A_1, A_2 – parameters of planetary gearsets I and II;

$z_9, z_{10}, z_{11}, z_{12}$ – numbers of teeth of gears 9, 10, 11, and 12.

- In range II of speed ratios, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{(1 + A_1) \frac{z_{11}}{z_{12}} i_z + \frac{z_9}{z_{10}}}{(1 + A_1 + A_2) \frac{z_{11}}{z_{12}} i_z + \frac{z_9}{z_{10}} (1 + A_2)} . \quad (29)$$

Hence:

$$A = (1 + A_1) \frac{z_{11}}{z_{12}} i_z, B = \frac{z_9}{z_{10}}, C = (1 + A_1 + A_2) \frac{z_{11}}{z_{12}}, D = \frac{z_9}{z_{10}} (1 + A_2), i_2 = \frac{1}{1 + A_2}, i_1 = \frac{1 + A_1}{1 + A_1 + A_2} ; \quad (30)$$

the other symbols have the meaning as above.

- In range III of speed ratios, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{A_3 i_z + \frac{z_8 \cdot z_{11}}{z_7 \cdot z_{12}} (1 + A_1)}{(A_3 - A_2) i_z + \frac{z_8 \cdot z_{11}}{z_7 \cdot z_{12}} (1 + A_1 + A_2)} \quad (31)$$

Hence:

$$A = A_3, B = \frac{z_8 \cdot z_{11}}{z_7 \cdot z_{12}} (1 + A_1), C = (A_3 - A_2), D = \frac{z_8 \cdot z_{11}}{z_7 \cdot z_{12}} (1 + A_1 + A_2), i_2 = \frac{1 + A_1}{1 + A_1 + A_2}, i_1 = \frac{A_3}{A_3 - A_2} \quad (32)$$

A_3 – parameter of planetary gearset III;

z_7, z_8 – numbers of teeth of gears 7 and 8.

the other symbols have the meaning as above.

- For the Toyota Prius transmission in the 1st gear, the transmission ratio is changed in accordance with the following relations:

$$i = \frac{(1 + A_1) i_r i_z}{A_1 i_z + i_r} \quad (33)$$

Hence:

$$A = (1 + A_1) i_r, B = 0, C = A_1, D = i_r, i_2 = 0, i_1 = \frac{(1 + A_1) i_r}{A_1}, \text{ and } i < 0 \text{ to } 1 > \text{ for } i_z < 0 \text{ to } 1 >$$

A_1 – parameter of the planetary gearset that splits the power into parallel flows, e.g. $A_1 = 3$;
 i_r – speed ratio of the planetary gearset that adds the power flows to each other, e.g. $i_r = 1$.

The relations presented above show that, within the range of speed ratios corresponding to the start of vehicle motion, the transmissions under consideration can be classified in one and the same structural group of parallel systems in spite of different engineering solutions, i.e. in the group where the CVT is connected with the mechanical transmission branch through a constant-ratio gear train situated at the system output.

In the Allison transmission operating in mode II, there is a connection in it through planetary (internal) gearsets. It can be easily noted that for the continuity of changes in the speed ratios to be maintained when the transmission is switched from mode I to II, the value of i_1 for mode I must be equal to that of i_2 for mode II; this takes place when $A_2 = A_3$. In addition to this, the vehicle speed values at which transmission components move with identical angular speeds and, thanks to that, the efficiency of operation of the transmission with parallel power flow reaches its maximum can be chosen by selecting appropriate parameters of the planetary gearsets.

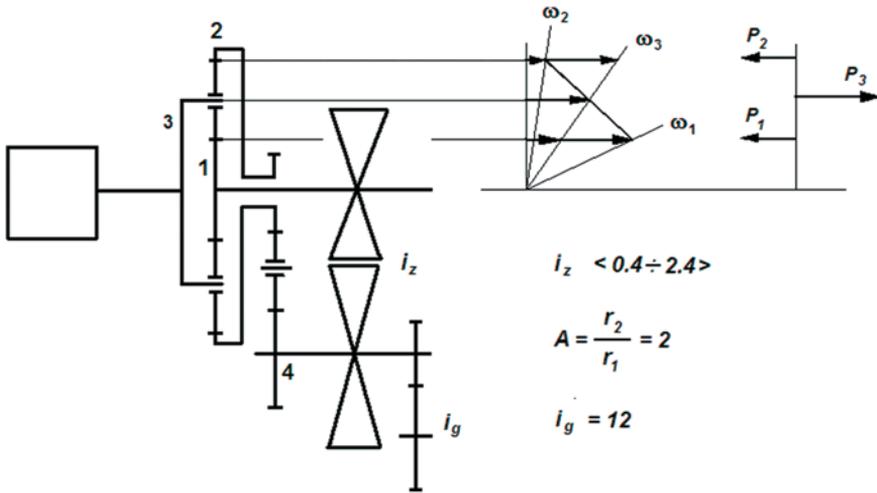
Similar relations take place in the TUL prototype [3].

This means that the method discussed above makes it possible to determine many basic parameters of the transmission system, inclusive of plotting the vehicle performance graph, even as early as at the stage of preparing a schematic diagram of the transmission to be designed.

3.3. Analysis of various connections of the CVT input shaft with a selected component of the planetary gearset

When taking into account conclusions of the findings summarized in Table 1, i.e. selecting the connection situated at the transmission output as the option that best corresponds to the transmission ratio ranges utilized in the phase of vehicle motion start, one should also pay attention to connecting the input shaft with the appropriate part of the planetary gearset, i.e. sun gear, planet carrier, or ring gear. An analysis of such connections has been presented below, with a friction gear being used as an example of the CVT unit.

Option I: CVT connected with the sun gear and engine connected with the planet carrier



In this option, the following kinematic relations hold:

$$\begin{aligned}
 \omega_1 + A_1 \omega_2 - (1 + A_1) \omega_3 &= 0 \\
 \omega_4 &= \omega_2 i_{42} \\
 \omega_4 &= \omega_1 i_z
 \end{aligned}
 \tag{34}$$

In result of transformations, the equation shown below is obtained:

$$\omega_4 \left(\frac{1}{i_z} + \frac{A_1}{i_{42}} \right) - (1 + A_1) \omega_3 = 0
 \tag{35}$$

Hence

$$i = \frac{\omega_4}{\omega_3} = \frac{i_{42} (1 + A_1) i_z}{A_1 i_z + i_{42}},
 \tag{36}$$

with the characteristic speed ratios of the parallel transmission (zero-power ratios) being

$$i_2 = \frac{B}{D} = 0, \quad i_1 = \frac{A}{C} = \frac{i_{42} (1 + A_1)}{A_1}
 \tag{37}$$

According to Table 1 and the speed ratios i_1 and i_2 presented above, the schematic diagram of the parallel system under consideration corresponds, in terms of its structure, to the system where the power flow branches are connected together by a constant-ratio gear train situated at the system output. Based on the equations given in Table 1, the following is obtained:

$$\frac{\omega_\alpha^*}{\omega_1^*} = k_\alpha \left(1 - \frac{i}{i_1} \right) = k_\alpha \left[1 - \frac{(1 + A_1) i_{42} i_z A_1}{(A_1 i_z + i_{42})(1 + A_1) i_{42}} \right] = k_\alpha \frac{i_{42}}{A_1 i_z + i_{42}} = \frac{\omega_1}{\omega_3}
 \tag{38}$$

According to the kinematics of the system:

$$\frac{\omega_1}{\omega_3} = \frac{(1 + A_1)i_{42}}{A_1 i_z + i_{42}}, \text{ i.e. } k_\alpha = 1 + A_1 \quad (39)$$

Based on Table 1:

$$\frac{M_\alpha^*}{M_1^*} = \frac{1}{k_\alpha} = \frac{1}{1 + A_1} = \frac{M_1}{M_3} \quad (40)$$

For the parameters of the CVT at the system output, we obtain:

$$\frac{\omega_\beta^*}{\omega_1^*} = k_\beta \frac{i}{i_1} = k_\beta \frac{(1 + A_1)i_{42} i_z A_1}{(A_1 i_z + i_{42})(1 + A_1)i_{42}} = \frac{\omega_4}{\omega_3} \Rightarrow k_\beta = \frac{(1 + A_1)i_{42}}{A_1} \quad (41)$$

$$\frac{M_\beta^*}{M_1^*} = \frac{1}{k_\beta} \left(\frac{i_1}{i} - 1 \right) = \frac{A_1}{(1 + A_1)i_{42}} \left[\frac{(1 + A_1)i_{42} (A_1 i_z + i_{42})}{A_1 (1 + A_1) i_{42} i_z} - 1 \right] = \frac{1}{(1 + A_1)i_z} = \frac{M_4}{M_3} \quad (42)$$

If the torque transmitted through the mechanical branch is taken into account, the total torque on shaft 4 will be:

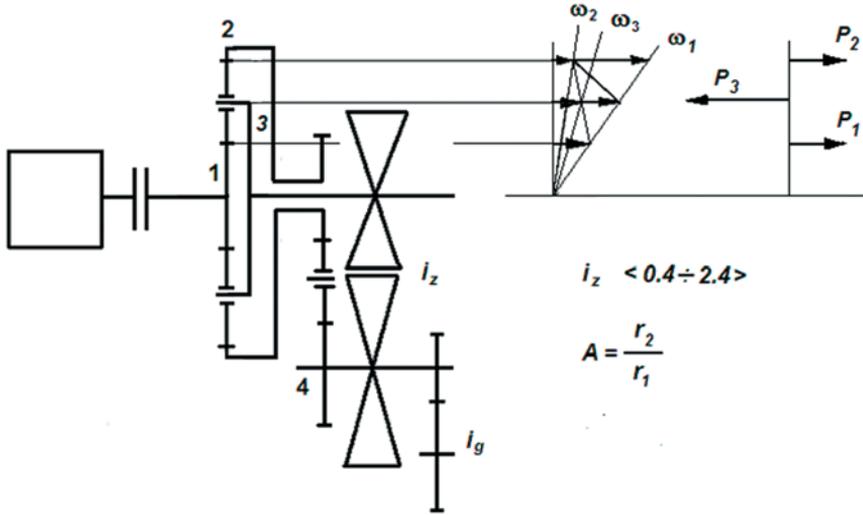
$$M_{4 \text{ r\u00f3w}} = \frac{M_3}{(1 + A_1)i_z} + \frac{M_3}{A_1 i_{42}} \quad (43)$$

The power flowing through the CVT unit (friction-type CVT in this case) is defined by the following:

$$\frac{N_z}{N_1^*} = 1 - \frac{i}{i_1} = 1 - \frac{(1 + A_1)i_{42} i_z A_1}{(A_1 i_z + i_{42})(1 + A_1) i_{42}} = \frac{i_{42}}{A_1 i_z + i_{42}} = \frac{N_{CVT}}{N_3} \quad (44)$$

According to the distribution of angular speeds of the planetary gearset components, when $\omega_1 = \omega_2 = \omega_3$, then the CVT speed ratio should reach the end of the CVT control range (where $i_z = i_{zmax}$). In such conditions, additionally, the planetary gearset, behaving as if it were blocked into a rigid unit, operates with its maximum efficiency. If the speed ratios of both the parallel power flow branches are assumed as equal to each other, e.g. $i_{42} = i_{zmax} = 2.4$ as well as $A = 1$ and $i_z \in \langle 0.4 \text{ to } 2.4 \rangle$, then $i_2 = 0$, $i_1 = 4.8$, $N_{CVT} \in \langle (0.857 \text{ to } 0.5)N_3 \rangle$ and $i \in \langle 0.68 \text{ to } 2.4 \rangle$; the value of speed ratio i is outside of the operating range of ratios of the parallel transmission. When a reduction gear with a ratio of $i_r = 0.42$ is introduced, it becomes possible to obtain $i \in \langle 0.28 \text{ to } 1 \rangle$, i.e. a range corresponding to that of a conventional system.

Option II: CVT connected with the planet carrier and engine connected with the sun gear



In option II, the following kinematic relations hold:

$$\begin{aligned} \omega_1 + A_1 \omega_2 - (1 + A_1) \omega_3 &= 0 \\ \omega_4 &= \omega_2 i_{42} \\ \omega_4 &= \omega_3 i_z \end{aligned} \tag{45}$$

In result of transformations, the equation shown below is obtained:

$$\omega_1 - \omega_4 \left(\frac{1 + A_1}{i_z} - \frac{A_1}{i_{42}} \right) = 0 \tag{46}$$

Hence

$$i = \frac{\omega_4}{\omega_1} = \frac{i_{42} i_z}{-A_1 i_z + i_{42} (1 + A_1)}, \tag{47}$$

with the characteristic speed ratios of the parallel system being

$$i_2 = \frac{B}{D} = 0, \quad i_1 = \frac{A}{C} = -\frac{i_{42}}{A_1} \tag{48}$$

It can be easily noted that there is a difference in the denominator of the expression defining the total speed ratio of the parallel transmission. This means that a value of i_z (denoted below by $i_{z \text{ nieokr}}$) exists at which indeterminacy and change of sign appears:

$$i_{z \text{ nieokr}} = \frac{i_{42} (1 + A_1)}{A_1} \tag{49}$$

At an assumption that $A_1 = 1$ and $i_z < 0.4 \text{ to } 2.4 >$, we obtain $i < 0.218 \text{ to } 2.4 >$, $i_2 = 0$, $i_1 = -2.4$, and $i_{z \text{ nieokr}} = -4.8$ (this value is outside of the operating range of ratios of the parallel transmission). When a reduction gear with a ratio of $i_r = 0.42$ is introduced, it becomes possible to obtain $i < 0.091 \text{ to } 1 >$, i.e. a range corresponding to that of a conventional system. The span of speed ratios of the transmission with parallel power flow is $R_{\text{rów}} = 11$, as against the span of $R_{\text{CVT}} = 6$ for the CVT unit.

According to Table 1 and the speed ratios i_1 and i_2 presented above, the schematic diagram of the parallel system under consideration corresponds, in terms of its structure, to the system where the power flow branches are connected together by a constant-ratio gear train situated at the system output. Based on the equations given in Table 1, the following is obtained:

$$\frac{\omega_\alpha^*}{\omega_1^*} = k_\alpha \left(1 - \frac{i}{i_1} \right) = k_\alpha \left[1 + \frac{i_{42} i_z A_1}{[i_{42}(1+A_1) - A_1 i_z] i_{42}} \right] = k_\alpha \frac{i_{42}(1+A_1)}{i_{42}(1+A_1) - A_1 i_z} = \frac{\omega_3}{\omega_1} \quad (50)$$

According to the kinematics of the system:

$$\frac{\omega_3}{\omega_1} = \frac{i_{42}}{-A_1 i_z + i_{42}(1+A_1)} \quad k_\alpha = \frac{1}{1+A_1} \quad (51)$$

Based on Table 1:

$$\frac{M_\alpha^*}{M_1^*} = \frac{1}{k_\alpha} = 1 + A_1 = \frac{M_3}{M_1} \quad (52)$$

For the parameters of the CVT at the system output, we obtain:

$$\frac{\omega_\beta^*}{\omega_1^*} = k_\beta \frac{i}{i_1} = k_\beta \frac{-i_{42} i_z A_1}{[-A_1 i_z + i_{42}(1+A_1)] i_{42}} = \frac{\omega_4}{\omega_1} \quad \Rightarrow \quad k_\beta = -\frac{i_{42}}{A_1} \quad (53)$$

$$\frac{M_\beta^*}{M_1^*} = \frac{1}{k_\beta} \left(\frac{i_1}{i} - 1 \right) = \frac{-A_1}{i_{42}} \left[\frac{-i_{42} [-A_1 i_z + i_{42}(1+A_1)]}{A_1 i_{42} i_z} - 1 \right] = \frac{1+A_1}{i_z} = \frac{M_4}{M_1} \quad (54)$$

If the torque transmitted through the mechanical branch is taken into account, the total torque on shaft 4 will be:

$$M_{4 \text{ rów}} = \frac{M_1(1+A_1)}{i_z} - \frac{M_1 A_1}{i_{42}} \quad (55)$$

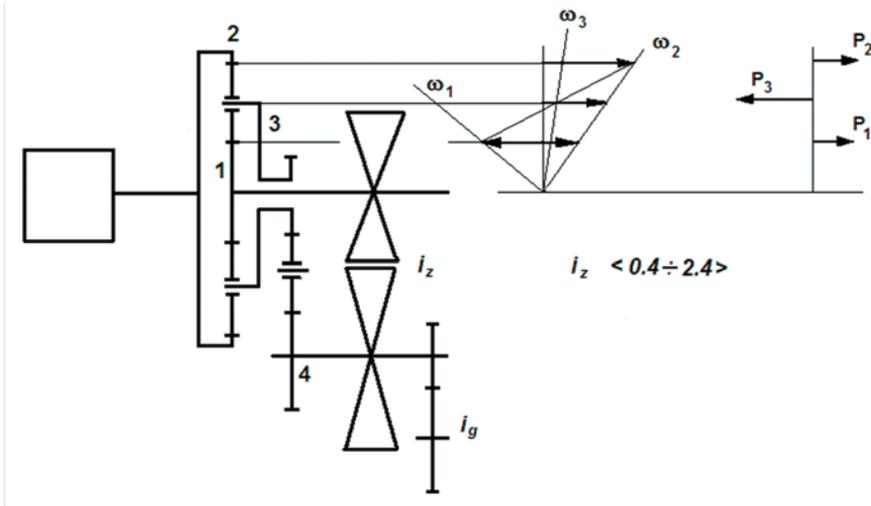
The power flowing through the CVT unit is defined by the following:

$$\frac{N_z}{N_1^*} = 1 - \frac{i}{i_1} = 1 - \frac{-i_{42} i_z A_1}{[-A_1 i_z + i_{42}(1+A_1)] i_{42}} = \frac{i_{42}(1+A_1)}{-A_1 i_z + i_{42}(1+A_1)} = \frac{N_{\text{CVT}}}{N_1} \quad (56)$$

According to the distribution of angular speeds of the planetary gearset components, when $\omega_1 = \omega_2 = \omega_3$ then the CVT speed ratio should reach the end of the CVT control range (where $i_z = i_{z \text{ max}}$). For the speed ratios assumed as above, we obtain $N_{\text{CVT}} < (1.09 \text{ to } 2)N_1 >$, which means that the phenomenon of power circulation takes place in the transmission

and the value of the circulating power may be even twice as high as that of the engine output power. The presence of the phenomenon of power circulation is also confirmed by vectors of linear velocity and forces acting on the planet gear.

Option III: CVT connected with the sun gear and engine connected with the ring gear



In option III, the following kinematic relations hold:

$$\begin{aligned} \omega_1 + A_1 \omega_2 - (1 + A_1) \omega_3 &= 0 \\ \omega_4 &= \omega_3 i_{43} \\ \omega_4 &= \omega_1 i_z \end{aligned} \tag{57}$$

In result of transformations, the equation shown below is obtained:

$$A_1 \omega_2 - \omega_4 \left(\frac{1 + A_1}{i_{43}} - \frac{1}{i_z} \right) = 0 \tag{58}$$

Hence

$$i = \frac{\omega_4}{\omega_2} = \frac{A_1 i_{43} i_z}{-i_{43} + i_z (1 + A_1)}, \tag{59}$$

with the characteristic speed ratios of the parallel system being

$$i_2 = \frac{B}{D} = 0, \quad i_1 = \frac{A}{C} = \frac{A_1 i_{43}}{1 + A_1} \tag{60}$$

At an assumption that $A_1 = 1$ and $i_z < 0.4 \text{ to } 2.4 >$, we obtain $i < -0.6 \text{ to } 2.4 >$. It can be easily noted that for $i_z = 1.2$, the denominator of the expression defining the total speed ratio of the parallel transmission becomes equal to zero. This has the meaning of a point of discontinuity, which complicates the issue of continuous controlling the transmission ratio.

Based on the equations given in Table 1, the following is obtained:

$$\frac{\omega_{\alpha}^*}{\omega_1^*} = k_{\alpha} \left(1 - \frac{i}{i_1} \right) = k_{\alpha} \left[1 - \frac{i_{43} i_z A_1 (1 + A_1)}{[i_z (1 + A_1) - i_{43}] A_1 i_{43}} \right] = k_{\alpha} \frac{-i_{43}}{i_z (1 + A_1) - i_{43}} = \frac{\omega_1}{\omega_2} \quad (61)$$

According to the kinematics of the system:

$$\frac{\omega_3}{\omega_1} = \frac{A_1 i_{43}}{-i_{43} + i_z (1 + A_1)} \text{ i.e. } k_{\alpha} = -A_1 \quad (62)$$

Based on Table 1:

$$\frac{M_{\alpha}^*}{M_1^*} = \frac{1}{k_{\alpha}} = -\frac{1}{A_1} = \frac{M_1}{M_2} \quad (63)$$

For the parameters of the CVT at the system output, we obtain:

$$\frac{\omega_{\beta}^*}{\omega_1^*} = k_{\beta} \frac{i}{i_1} = k_{\beta} \frac{i_z (1 + A_1)}{[i_z (1 + A_1) - i_{43}]} = \frac{\omega_4}{\omega_2} \Rightarrow k_{\beta} = \frac{A_1 i_{43}}{1 + A_1} \quad (64)$$

$$\frac{M_{\beta}^*}{M_1^*} = \frac{1}{k_{\beta}} \left(\frac{i_1}{i} - 1 \right) = \frac{1 + A_1}{A_1 i_{43}} \left[\frac{A_1 i_{43} [i_z (1 + A_1) - i_{43}]}{(1 + A_1) A_1 i_{43} i_z} - 1 \right] = -\frac{1}{A_1 i_z} = \frac{M_4}{M_2} \quad (65)$$

If the torque transmitted through the mechanical branch is taken into account, the total torque on shaft 4 will be:

$$M_{4\text{row}} = -\frac{M_2}{A_1 i_z} + \frac{M_2 (1 + A_1)}{A_1 i_{43}} \quad (66)$$

The power flowing through the CVT unit is defined by the following:

$$\frac{N_z}{N_1^*} = 1 - \frac{i}{i_1} = 1 - \frac{-i_{43} i_z A_1 (1 + A_1)}{[i_z (1 + A_1) - i_{43}] A_1 i_{43}} = \frac{i_{43}}{i_z (1 + A_1) - i_{43}} = \frac{N_{CVT}}{N_1} \quad (67)$$

For the speed ratios assumed as above, we obtain $N_{CVT} < (1.5 \text{ to } -1) N_1 >$, which means that the phenomenon of power circulation may take place in the transmission and that the transmission operation shows a point of discontinuity at $i_z = 1.2$, i.e. within the working range of CVT ratios. The presence of the phenomenon of power circulation is also confirmed by vectors of linear velocity and forces acting on the planet gear.

To sum up, option I of the system architecture should be considered most advantageous as being free of power circulation.

Proposed parallel power flow systems with a conventional gearbox

The benefits that accrue from splitting the power into two parallel flows may also become easily visible if a conventional gearbox is put in place of the CVT. It is obvious that if a planetary gearset is used to split the IC engine output power, the distribution of the

partial power flows depends on geometric parameters of the gearset (A) – see Fig. 8. Therefore, the conventional gearbox may be designed for an appropriately lower power capacity. Thanks to this, the dimensions and mass of such a gearbox may be reduced or even materials of lower strength, e.g. plastics, may be used. For example, as it has been shown in Fig. 8, if $A = 4$ then the torque on the input shaft of the conventional gearbox may be reduced to one fifth; in consequence, the gearbox length may be reduced and another gear synchronization concept, such as e.g. that shown in Fig. 9, may be sought.

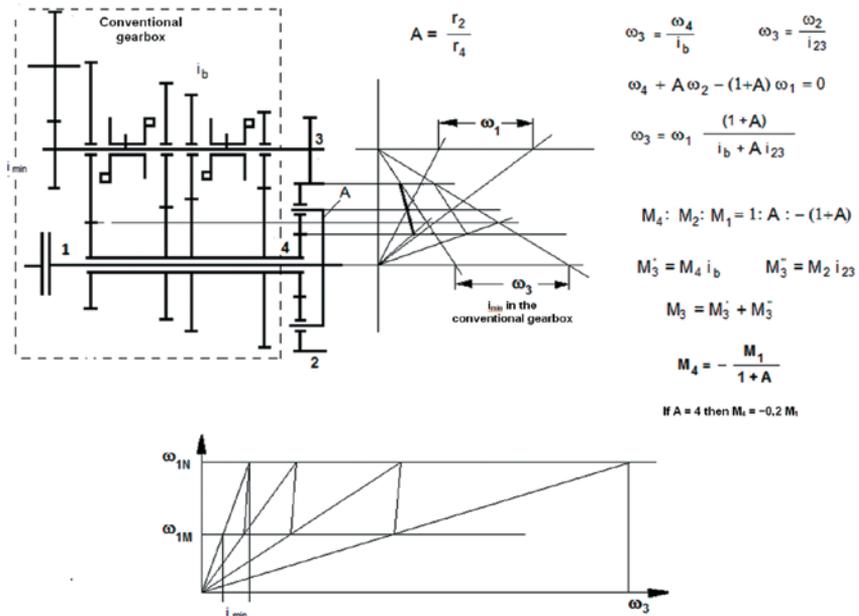


Fig. 8. Values of the parameters of a transmission with parallel power flow and a conventional gearbox: i_b – speed ratio of gear "b"

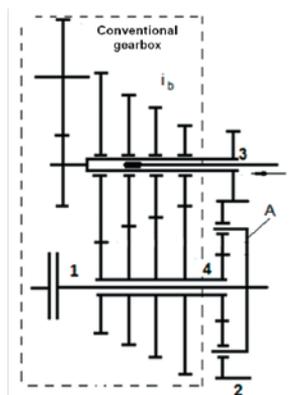


Fig. 9. Schematic diagram of a transmission with parallel power flow and a conventional gearbox, with a changed concept of the gear synchronization system

4. Recapitulation

A common feature of the motor vehicle drive systems under analysis is the fact that the transmission input shaft driven by the IC engine rotates with a constant angular speed in spite of varying output shaft speed corresponding to the vehicle speed. This makes it possible to reduce the engine operation area to a very narrow part of the engine performance range, which is essential for vehicles operated in urban traffic conditions. Thanks to the fact that the function of controlling the vehicle speed has been shifted from the engine to the transmission, the latter may also be used to transmit energy in the opposite direction during the vehicle braking process for the energy to be recuperated and stored in an electric storage battery. The transmissions capable to perform such a function are fundamental for hybrid drive systems of motor vehicles.

The presented method of analysing the vehicle transmission systems with parallel power flow is a useful tool for determining the kinematic and dynamic properties of such systems even as early as at the project design stage. The essence of this method lies in determining the values of two characteristic transmission speed ratios i_1 and i_2 of a compound system to ascertain whether the transmission under consideration is or is not a parallel power flow system. This method may also be used to explore new existing designs.

The presented analysis of the vehicle drive system designs enables the reader to familiarize himself/herself with the method proposed and to check his/her practical skill in using this method.

The full text of the Article is available in Polish online on the website <http://archiwummotoryzacji.pl>.

Tekst artykułu w polskiej wersji językowej dostępny jest na stronie <http://archiwummotoryzacji.pl>.

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