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UNCERTAINTY OF THE ESTIMATION OF MODEL PARAMETER VALUES IN THE ANALYSIS OF PEDESTRIAN THROW

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Summary

The objective of the work reported in the article was to analyse the influence of the uncertainty of estimating the values of parameters of selected mathematical models, built to describe the relations between the pedestrian throw distance and the launch velocity gained by the pedestrian in result of being struck by the front part of the vehicle body. Within the work, the impact of changes in the pedestrian launch angle on the pedestrian launch velocity was investigated and the problems connected with choosing the value of the coefficient of friction between pedestrian's clothing and the road surface were shown. The analysis presented may be useful in the reconstruction of an accident where a motor vehicle ran into a pedestrian.

Keywords: vehicle/pedestrian collision, pedestrian launch angle, coefficient of friction between pedestrian's clothing and the road surface

Notation:

- g – acceleration of gravity
- h – apex height of the trajectory of pedestrian's centre of mass in the flight phase
- h_0 – height of the point of the first contact between the vehicle body and the pedestrian
- S – total pedestrian throw distance
- vs – collision velocity (velocity of the motor vehicle at the instant of hitting the pedestrian)
- v_p – pedestrian launch velocity (gained by the pedestrian in result of being struck by the front part of the vehicle body)
- θ – pedestrian launch angle
- μ – coefficient of friction between pedestrian's clothing and the road surface

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1. Pedestrian throw models

The determination of the pedestrian launch velocity at the instant of a collision with a motor vehicle from the pedestrian throw distance is a standard element of the reconstruction of a vehicle/pedestrian collision accident. There are two types of the models used for the reconstruction of a vehicle/pedestrian collision: empirical models and models based on the laws of mechanics. The empirical models are most often presented in the form of a simple formula making it possible to determine the value of the collision velocity v_s of the motor vehicle at the instant of hitting the pedestrian, based in most cases on the estimation of pedestrian throw distance S. The applicability of the empirical models is limited to the cases with well-defined scenarios and their accuracy is relatively low, of the order of ±10 km/h [11].

Results that would be more reliable may be obtained by using models based on the laws of mechanics (mathematical models). Such models make it possible to determine the pedestrian launch velocity v_p, gained by the pedestrian at the beginning of the launch phase, from the estimated values of the input parameters of the model.

The main source of the uncertainty of the results obtained from the analysis of the mathematical model is the discrepancy between the simplifying assumptions made at the construction of the theoretical model and the real road situation.

Most often, the schematic diagram presented in Fig. 1 is adopted to represent the course of the pedestrian throw. In particular, an assumption is made that the impact against the pedestrian, treated as a material particle, took place in the longitudinal symmetry plane of the vehicle. Even if the pedestrian was moving when being struck by the vehicle, his/her velocity was very low in comparison with the vehicle velocity and, therefore, it may be ignored. If the vehicle collision velocity v_s is high enough, the trajectory of pedestrian's body consists of the flight phase and the phase of movement of the pedestrian on the road surface (with disregarding the short time of the pedestrian being in contact with the vehicle body).

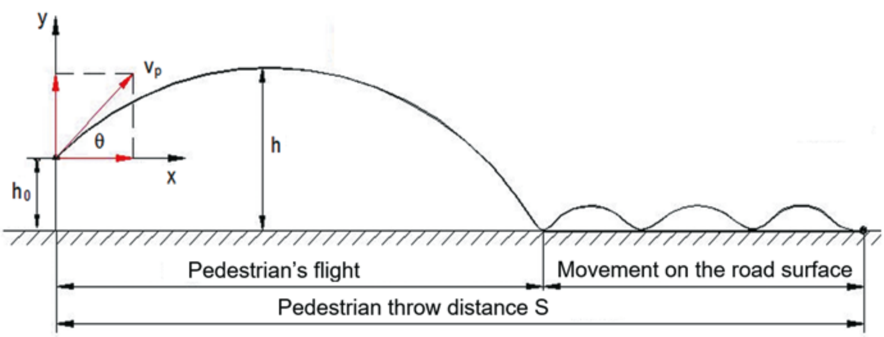

Fig. 1. Schematic diagram of the pedestrian throw process

The flight phase is analysed without taking into account the air resistance. The pedestrian, struck at a height h_0 and at a launch angle θ, initially flies to reach a height h and then falls onto the road surface. The angle θ depends on vehicle body type, location of the centre of pedestrian's mass, and height h_0 of the point of the first contact between the vehicle body and the pedestrian. The movement on the road surface may have the form of sliding or tumbling to rest. In this phase, the resistance of pedestrian's body to motion depends on the coefficient of friction between pedestrian's clothing and the road surface.

It is worth pointing out that the pedestrian launch velocity v_p, caused by the impact of the vehicle body, is lower than the collision velocity v_s of the motor vehicle ($v_p < v_s$). This is due to partial conversion of the kinetic energy of the vehicle into vehicle body deformation work and energy dissipation resulting from the fact that pedestrian's body is not a perfectly rigid solid (the collision is not perfectly elastic). Such a phenomenon may be taken into account by using the notion of "coefficient of rebound" κ (coefficient of rebound efficiency, defined similarly as the coefficient of restitution for colliding solids, i.e. $\kappa = v_p/v_s$). The estimated values of this coefficient may be found in the literature, e.g. [25].

There are many mathematical models representing the pedestrian throw process. Some of those being most popular have been given in Table 1.

Table 1. Popular pedestrian throw models

Item	Model	Source
1.	$v_{pS} = \dfrac{\sqrt{2\mu gS}}{\cos\theta + \mu\sin\theta}$	Searle, J. A.; Searle, A. (1983) [21]
2.	$v_{pS1} = \dfrac{\sqrt{2\mu g(S+\mu h_0)}}{\cos\theta + \mu\sin\theta}$	Searle, J. (1993) [19]
3.	$v_{pA} = \dfrac{-\mu gt + \sqrt{(\mu gt)^2 + 2\mu gS}}{\cos\theta}$ $t = \dfrac{\sqrt{2g}}{g}\left(\sqrt{(h-h_0)} + \sqrt{h}\right)$	Aronberg, R. (1990) [1]
4.	$v_{pW} = \dfrac{\sqrt{2\mu g(S-\mu h_0)}}{\cos\theta + \mu\sin\theta}$	Wood, D. P. (1991) [26]

2. Estimation of the values of individual parameters in the pedestrian throw models

A standard element of the reconstruction of an accident with a vehicle/pedestrian collision is the determination of the pedestrian launch velocity v_p at the instant of collision with a motor vehicle. Table 1 shows equations having the form $v_p = f(S, \mu, \theta, h0, h, g)$. Let us analyse the possibilities of estimating the values of parameters S, μ, θ, $h0$, h, and g. The value of g (acceleration of gravity) is known. The height of the point of the first contact between the vehicle body and the pedestrian may be estimated on the grounds of the vehicle body type. The total pedestrian throw distance S may also be estimated, based on the information of the specific road incident. The Aronberg model includes parameter h, defined as the apex height of the trajectory of pedestrian's centre of mass in the flight phase. The value of this parameter may be estimated with the use of the equations known from mechanics and describing the dynamics of a material particle motion representing the pedestrian's flight phase (in particular, the vertical component of this motion). An important parameter is the pedestrian launch angle θ. Its value chiefly depends on the vehicle body type. In the case of pontoon- or box-type bodies, the pedestrian launch angle θ is defined by the tilt of the front part of the vehicle body and is equal to zero or does not exceed several degrees. This angle is very difficult to estimate in the case of wedge-shaped or trapezoidal car bodies. Fig. 2 shows the impact of the pedestrian launch angle θ on the pedestrian launch velocity v_p in the Searle and Wood models (items 1, 2, and 4 in Table 1). The calculations were carried out for the pedestrian launch angle values varying within a range of $1° \leq \theta \leq 85°$ (0.017 rad $\leq \theta \leq 1.484$ rad). The other values of the model parameters may be estimated from the data collected for a specific collision. In the calculations presented, the model parameter values were adopted as $S = 16$ m, $\mu = 0.7$, $h_0 = 1$ m, $h = 1.5$ m, $g = 9.81$ m/s^2.

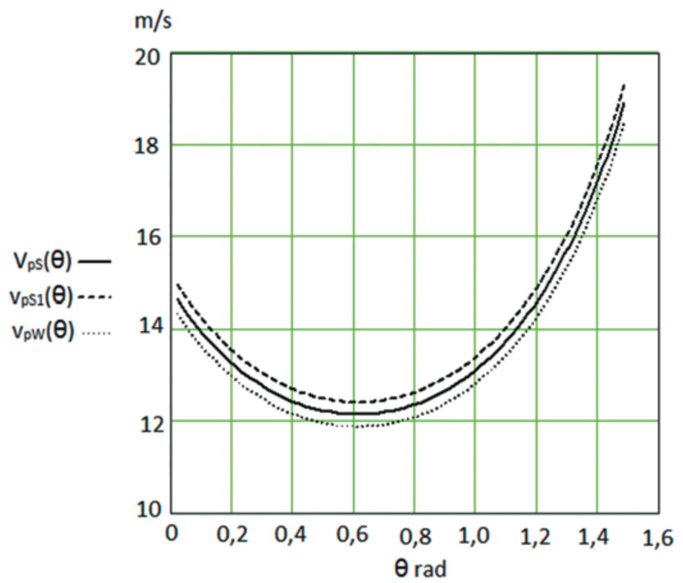

Fig. 2. Pedestrian launch velocity vs. pedestrian launch angle curves:
v_{pS} – model by Searle, J. A., and Searle, A. (1983);
v_{pS1} – model by Searle, J. (1993);
v_{pW} – model by Wood, D. P. (1991) (see Table 1)

The curves representing the calculation results obtained from the Searle and Wood's models are very similar to each other. This is related to the fact that in each of these models, the expression in the denominator (where angle θ is present) is identical. Interesting may be the observation that the mathematic formulas adopted in both of the Searle's models and in the Wood's model have a minimum, which is close to the value of θ = 0.6 rad (34.4 °). In the Aronberg model, the denominator value cos $\theta \rightarrow 0$ when $\theta \rightarrow 90$ °; hence, the calculated pedestrian launch velocity value $v_{pA} \rightarrow \infty$. Obviously, such a calculation result only reflects the imperfection of the mathematical model adopted and it has nothing to do with the actual pedestrian launch velocity. The most difficult thing is to estimate the value of arameter μ, which defines the friction between pedestrian's clothing and the road surface. This is one of the fundamental parameters necessary for the reconstruction of a vehicle/pedestrian collision accident. This parameter, used for pedestrian throw calculations, has been for decades one of the main sources of controversies between authors of publications dedicated to investigations on the motor vehicle/pedestrian collision accidents. Fig. 3 shows the interrelation between the pedestrian launch velocity v_p and the coefficient of friction μ. The calculations were carried out for the friction coefficient values varying over a range of $0.1 \leq \mu \leq 1.2$ and for $\theta \approx 28$ °. In publication [7], the authors have made a remark that this is a typical value of the pedestrian launch angle. The other parameter values were as specified above, i.e. S = 16 m, μ = 0.7, h_0 = 1 m, h = 1.5 m, and g = 9.81 m/s^2.

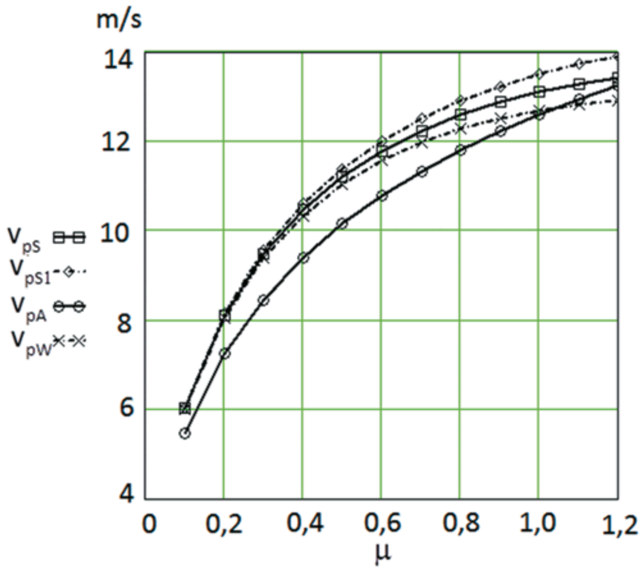

Fig. 3. Pedestrian launch velocity vs. coefficient of friction between pedestrian's clothing and the road surface:
v_{pS} – model by Searle, J. A., and Searle, A. (1983);
v_{pS1} – model by Searle, J. (1993);
v_{pA} – model by Aronberg, R. (1990);
v_{pW} – model by Wood, D. P. (1991) (see Table 1).

Based on the curves presented in Fig. 3, it can be easily noticed that the impact of the value of μ on velocity v_p within the interval of ~ $0.5 \leq \mu \leq 1.2$ is lower by almost a half than that within the interval of $0.1 \leq \mu \leq \sim 0.5$, which is related to the slope of the curve. A good measure to evaluate the impact of the value of parameter μ on the function value may also be the sensitivity coefficient W, defined as the partial derivative of the function with respect to this parameter. In the case under consideration, $W_\mu = \partial v_p(\mu)/\partial\mu$. The value of this coefficient is proportional to the slope of the line tangent to the curve $v_p(\mu)$ at the point in question; thus, it provides information about the impact of μ on vp. The sensitivity function curves shown in Fig. 4 have been plotted to represent the partial derivatives $W_{\mu S} = \partial v_{pS}(\mu)/\partial\mu$, $W_{\mu S1} = \partial v_{pS1}(\mu)/\partial\mu$, $W_{\mu A} = \partial v_{pA}(\mu)/\partial\mu$, and $W\mu W = \partial v_{pW}(\mu)/\partial\mu$ for the mathematical models listed in Table 1.

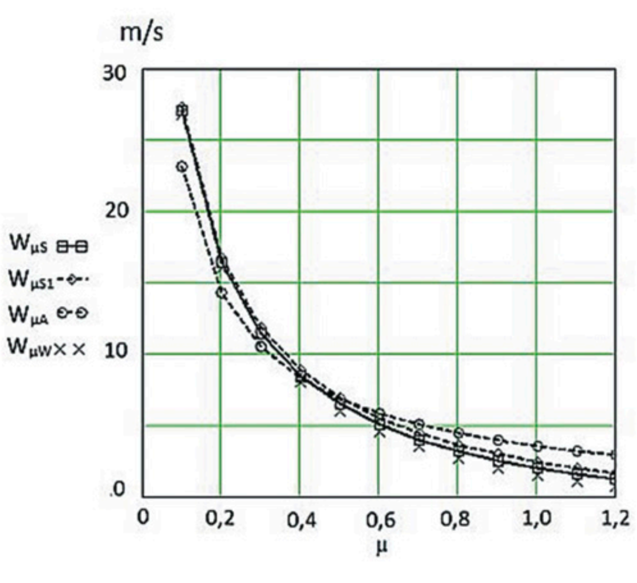

Fig. 4. Curves representing the sensitivity functions W determined for the mathematical models shown in Table 1:
v_{pS} – model by Searle, J. A., and Searle, A. (1983);
v_{pS1} – model by Searle, J. (1993);
v_{pA} – model by Aronberg, R. (1990);
v_{pW} – model by Wood, D. P. (1991) (see Table 1).

The results obtained confirm the conclusion formulated above on the grounds of Fig. 3: the impact of parameter μ on velocity vp within the interval of $\sim 0.5 \leq \mu \leq 1.2$ is far less than that within the interval of $0.1 \leq \mu \leq \sim 0.5$. Obviously, the findings presented are exclusively applicable to the mathematical models shown in Table 1 and they should not be automatically extrapolated to all the pedestrian throw models. Many different values and ranges of values of parameter μ are given in the literature. Table 2 shows the results found in various sources, arranged in the order of appearance in publications. It can be easily noticed that the discrepancies between them are very wide and it is difficult to formulate any criterion of selection of the μ values.

Table 2. Values of the coefficient of friction between the pedestrian and the road surface

Item	Coefficient of friction μ	Road surface / pedestrian's clothing	Source
1.	0.40-0.75	?	Severy, D. (1966)
2.	0.51-0.61	Dry road surface	Rychter, W. (1973)
3.	0.52-0.59	Dry road surface	Rychter, W. (1973)
4.	0.61-1.02	?	Löhle, U. (1975)
5.	1.1	?	Collins, J. C.; Morris, J. L. (1979)
6.	0.66	Dry and wet asphalt	Searle, J.; Searle, A. (1983)
7.	0.79	Grass	Searle, J.; Searle, A. (1983)
8.	0.61-0.71	Dry road surface	Becke, M.; Golder, U. (1986)
9.	0.46-0.56	Wet road surface	Becke, M.; Golder, U. (1986)
10.	0.6	?	Batista, M. (2008)
11.	0.45-1.2	?	Rotim, F. (1989)
12.	0.80	Tumbling	Hill, G. S. (1994)
13.	0.39-0.87	?	Wood, D.; Simms, C. (2000)
14.	0.7-1.2	Dry asphalt, tumbling	Happer, A. et al, (2000)
15.	0.45-0.72	Dry asphalt, sliding	Happer, A. et al, (2000)
16.	0.37-0.75	Dry and wet asphalt	Happer, A. et al, (2000)
17.	0.59-0.85	Asphalt with anti-slip surface coating / pedestrian wearing normal clothing	Hague, D. J. (2001)
18.	0.54-0.65	Asphalt with anti-slip surface coating / pedestrian wearing nylon clothing	Hague, D. J. (2001)
19.	0.74	?	Han, I.; Brach, R. M. (2001)
20.	0.73-0.78	Different types of pedestrian's clothing (except nylon)	Han, I.; Brach, R. M. (2001)
21.	0.61	Nylon clothing	Han, I.; Brach, R. M. (2001)
22.	0.31-0.41	Wet asphalt	Fugger, T. F. J. et al. (2002)
23.	0.45-0.55	?	Toor, A.; Araszewski, M. (2003)
24.	0.13-0.76	?	Batista, M. (2008)

The difficulties in correctly estimating the coefficient of friction between pedestrian's clothing and the road surface (sometimes referred to as "coefficient of resistance") arise from the fact that the motion of pedestrian's body on the road surface may have the form of sliding, tumbling, or both. The tumbling may be explained as follows: the first contact of pedestrian's body with the ground is followed by next short phases, during which the body is alternately tossed into the air by existing road surface irregularities (potholes, ruts, etc.) and falls down again. The measurements carried out with the use of dummies or cadavers most often represent the sliding, in result of which the friction coefficient values are lower than they actually are during a real accident, when the pedestrian's body having fallen onto the road surface is partly sliding and partly tumbling. Some researchers claim that the value of the coefficient of friction depends on the velocity of pedestrian's body sliding along the road surface, but there are no unequivocally ascertained data to confirm such a statement. The data presented in Table 2 have been shown in a graphical form in Fig. 5.

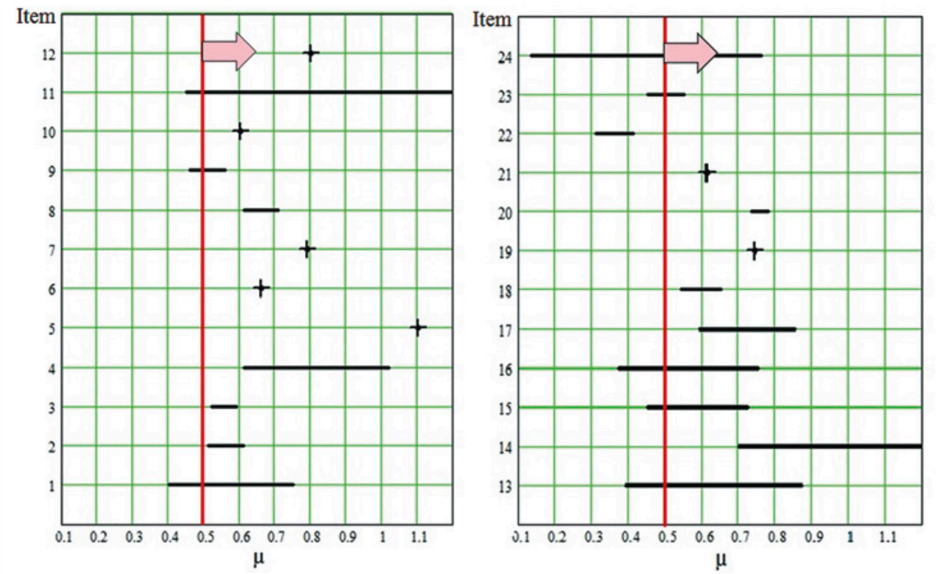

Fig. 5. Graphical presentation of variations in the μ values given in Table 2. The values on the vertical axis correspond to the "Item" values in the first column of the Table

When analysing the data given in Table 2 and Fig. 5, one can easily notice that the coefficient of friction may take values from the interval of 0.13-1.2. Simultaneously, the results specified by various authors should be compared with each other with a dose of scepticism. It is not certain, how some of these values were obtained. It is quite likely that in some cases, the specified value of the coefficient of friction was averaged over the overall pedestrian throw distance, which included the pedestrian's flight phase. During the flight, the coefficient of friction is equal to zero, but the air resistance, although not taken into account in the mathematical model, is present and has an impact on a reduction in the velocity of the pedestrian's body motion. The big differences in the values of the coefficient of friction between pedestrian's body and the road surface are also related to such factors as the lack of adequate information about the stiffness of the human skeletal system and the damping characteristics of pedestrian's body and clothing. The other factors that may have an influence on this issue include position of pedestrian's body in relation to the vehicle body solid at the instant of the collision, dimensions (especially of the position of the centre of mass) of the pedestrian, as well as direction and velocity of pedestrian's motion. Now, a question arises what strategy should be adopted to select the μ coefficient value. As it can be seen in Fig. 5, most of the values suggested in various publications (i.e. over 60-70 % of them) fall within the range of $\sim 0.5 \leq \mu \leq 1.2$. If a μ value is taken from this range for the calculations, then the difference between the v_p values for μ = 0.5 and for $\mu \approx 1.0 \div 1.2$ is about 3 m/s (10.8 km/h), based on the curves presented in Fig. 3; hence, the maximum error of the estimation of velocity v_p should be considered significant. For the range of $0.1 \leq \mu \leq \sim 0.5$ (about 30 % of the test results), the difference between the outermost v_p values is about 5 m/s (18 km/h), according to Fig. 3. The errors of this size suggest low practical usability of the calculations like these for the reconstruction of a vehicle/pedestrian collision accident. The analysis presented shows the reasonability of assuming that the uncertainty of estimation of the actual friction coefficient value significantly affects the results of calculation of the pedestrian launch velocity.

3. Final remarks

The large number of vehicle/pedestrian collision accidents induces a trend towards careful examining of the models that are to describe the vehicle/pedestrian collision. The reliability of the calculation results obtained with the use of such models is unsatisfactory. This is because many important factors are not taken into account in the mathematical description of a real road traffic event. The simplifying assumptions may lead to unrealistic calculation results. As an example: in the Aronberg model, the calculated pedestrian launch velocity value approaches infinity ($vpA \rightarrow \infty$) when the pedestrian launch angle value approaches 90 ° ($\theta \rightarrow 90$ °). The calculation results like this have nothing to do with the actual pedestrian launch velocity; instead, they only reflect the imperfection of the mathematical model adopted. It is very difficult to estimate correctly the values of the input parameters for the mathematical models used in practice. A particularly wide range of variability can be observed for the results of estimating the values of the coefficient μ of friction between pedestrian's clothing and the road surface. The data presented in Table 2 and in Fig. 5 show that the estimated values of this coefficient may vary within a range of $0.1 \leq \mu \leq 1.2$, which results in a big error of estimating the velocity v_p gained by the pedestrian in result of being struck by the front part of the vehicle body. As an example: at an assumption of $\mu \leq 0.5$, the v_p estimation error may be expected to reach a value even of the order of 18 km/h. The observations presented herein should facilitate the making of a decision as regards the question what value of this coefficient may be reasonably adopted at the reconstruction of a vehicle/pedestrian collision.

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