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# THE USE OF MULTI-SPEED MECHANICAL TRANSMISSION IN ELECTRIC DRIVES

## ZASTOSOWANIE WIELOBIEGOWEJ PRZEKŁADNI MECHANICZNEJ W NAPĘDZIE ELEKTRYCZNYM

**ARKADIUSZ HAJDUGA<sup>1</sup>, ANNA KIERACIŃSKA<sup>2</sup>**

Warsaw University of Technology  
Automotive Industry Institute (PIMOT)

### Summary

Electric vehicles are now a viable alternative to their combustion counterparts. Despite their obvious advantages, one prominent disadvantage is their limited driving range, which strongly depends on the operating conditions. Therefore, it seems purposeful to adapt the configuration of the electric drive to obtain its high efficiency regardless of the operating conditions of the drive, which will in turn reduce the energy consumption without increasing the energy storage capacity of the electrochemical battery.

This article will describe the impact of the applied multi-speed transmission on the electric power consumption. The use of multi-speed transmission solutions in electric drives allows to adapt the electrical machine's operating parameters to the load conditions to keep its operating efficiency as close to high-performance parameter range as possible. A computer simulation study has been carried out to verify the influence of applying an additional element of the drive train system on the energy consumption. The study compares two setups with different transmission types used. Namely, one configuration features a constant-ratio transmission drive whilst the other a transmission allowing the selection of three ratios. The results obtained on the basis of computer simulation confirmed the positive impact of the multi-speed transmission on energy consumption

<sup>1</sup> Warsaw University of Technology, the Faculty of Automotive and Construction Machinery Engineering, the Institute of Vehicles, ul. Narbutta 84, 02-791 Warsaw, Poland; e-mail: [arkadiusz.hajduga@simr.pw.edu.pl](mailto:arkadiusz.hajduga@simr.pw.edu.pl)

<sup>2</sup> Automotive Industry Institute (PIMOT), ul. Jagiellońska 55, 01-301 Warsaw, Poland; e-mail: [a.kieracińska@pimot.eu](mailto:a.kieracińska@pimot.eu)

in both urban and sub-urban operating conditions, whereas in the first case this reduction was significant. This clearly indicates that for vehicles operated in cities it is preferable to use electric drives equipped with a multi-speed transmission.

**Keywords:** electric drive, multi-speed transmission, driving range, simulation studies, mathematical modelling

## Streszczenie

Pojazdy z napędem elektrycznym są dziś realną alternatywą dla pojazdów spalinowych. Pomimo ich niewątpliwych zalet istotną niedogodnością jest ich ograniczony zasięg jazdy, który silnie zależy od warunków eksploatacji. Celowe, zatem wydaje się dostosowanie konfiguracji napędu elektrycznego w taki sposób, aby bez względu na warunki eksploatacji napęd uzyskiwał możliwie wysokie sprawności, co wpłynie na zmniejszenie zużycia energii, bez konieczności zwiększania pojemności energetycznej baterii elektrochemicznej.

W artykule przedstawiono wpływ zastosowania przekładni wielobiegowej na zużycie energii elektrycznej. Dzięki wyposażeniu napędu elektrycznego w przekładnię wielobiegową istnieje możliwość dopasowania parametrów pracy maszyny elektrycznej do warunków obciążenia w taki sposób, by sprawność jej pracy znajdowała się możliwie blisko obszaru najwyższej sprawności. W celu weryfikacji wpływu zastosowania dodatkowego elementu w układzie napędowym na zużycie energii przeprowadzono komputerowe badania symulacyjne. W badaniach porównano dwie struktury różniące się zastosowaną przekładnią, tj. w pierwszej konfiguracji zastosowano przekładnię o stałym przełożeniu zaś w drugiej konfiguracji przekładnię pozwalającą na wybór trzech przełożeń. Uzyskane na drodze symulacji komputerowych wyniki potwierdziły pozytywny wpływ przekładni wielobiegowej na zużycie energii zarówno w miejskich jak i pozamiejskich warunkach eksploatacji, przy czym w pierwszym przypadku zmniejszenie to było znaczące. Wskazuje to wyraźnie, że w przypadku pojazdów eksploatowanych w miastach korzystne jest zastosowanie napędu elektrycznego wyposażonego w przekładnię wielobiegową.

**Słowa kluczowe:** napęd elektryczny, przekładnia wielobiegowa, zasięg jazdy, badania symulacyjne, modelowanie matematyczne

## 1. Introduction

Electric vehicles are becoming increasingly popular and are an alternative for vehicles equipped with a classic combustion engine. An increasing array of models featuring an electric drive is entering the market every year. In addition, note that these models are not fitted with a classic setup of a primary combustion-drive with an auxiliary, adapted electric drive. Conversely, the designs are modern solutions dedicated exclusively for electric drive applications. The majority of these vehicles use an electrochemical battery as the drive power source, but also models powered by a fuel cell can be found. Some of the wide range of models available are the following: Mitsubishi i-MiEV [1] and Tesla Roadster [2]. The makes listed below are also extensively involved in the electric vehicles market development: Renault (Zoe, Twizy, Kangoo Z.E., Fluence Z.E., Kangoo Express Z.E.) [3-7], Nissan (Leaf, e-NV200) [8, 9], BMW (i3, ActiveE) [10, 11], Chevrolet (Volt, Spark EV) [12, 13]. However, also other leading manufacturers are becoming increasingly involved in the development of electric vehicles, having launched the following models: Fiat (500E [14]), Honda (FCX,

FIT [15, 16]), Mercedes-Benz (SLS AMG Coupé Electric Drive, Vito E-CELL, B Class Electric Drive [17-19]), Ford (Transit Connect EV, Focus Electric [20, 21]), Smart (Smart Electric Drive [22]), Toyota (iQ EV, RAV4 EV [23, 24]), Volkswagen (e-golf, e-up! [25, 26]), Citroen (Berlingo Electric, C-Zero [27, 28]).

As can be seen, the development of electric road transport solutions is not only limited to small urban vehicles, which, because of their operating conditions are ideal for the introduction of this type of drive, but is also applied for sports vehicles, limousines and even light commercial vehicles. Therefore, it can be assumed that an electric drive can be applied in virtually any car type, with the exception of heavy goods vehicles.

Despite the variety of vehicle types, as well as different requirements of particular types, the design of the electric drive is identical in each case. Electrical power, necessary to drive the vehicle, is stored in the electrochemical battery, with Li-Ion battery being the most common type (next to a fuel cell, as in the case of Mercedes-Benz Vito E-CELL [29]). The electrical energy is transmitted through the control unit (inverter) to the electric machine (usually with permanent magnets of BLDC or PMS type), where it is converted into mechanical energy, that in turn is transferred through a mechanical transmission to the driving wheels. This configuration results directly from the traction characteristics of electric machine-control system.

## 2. The Analysis of the Electric Drive Configuration on the Operating Conditions of the Electrical Machine

Figure 1 shows the theoretical characteristics of an electric machine controlled by an inverter with the use of the PWM (Pulse Width Modulation) method.

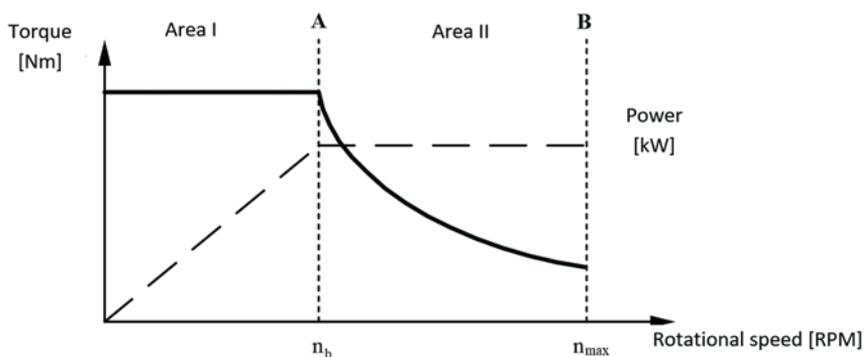


Fig. 1. The characteristics of the electrical machine controlled by PWM (Pulse Width Modulation) Method.

The Characteristics of the Electric Machine Controlled by PWM (Pulse Width Modulation) Method, shown in Figure 1, allows us to distinguish two areas. The first area clearly indicates

that the torque of the electric machine is constant and the mechanical power produced by it increases in line with the increase of its rotational speed. Conversely, in the second area, it is the mechanical power produced by the electric machine that is constant and its torque decreases hyperbolically with the increasing rotational speed, to maintain the power of the electric machine within this RPM range at constant level.

In the first regulation zone the rotational speed of the electric machine is relatively small (0 to  $n_b$ ). In the case of a vehicle drive train system this situation corresponds to driving at low speeds also characterized by frequent acceleration and braking. Therefore, it can be stated that the demand for electric machine torque is mainly related to inertia force of the vehicle that requires the electric machine to supply a relatively high torque. It should be noted here that the torque and power of the electric machine are available from zero rotational speed. In the second regulation zone ( $n_b - n_{max}$ ) the motion of the vehicle becomes more stabilized, and there is no need for implementing higher accelerations. Therefore, the electric machine torque demand is chiefly related to rolling and aerodynamic resistance of the vehicle. The torque of the electric machine in this case can achieve much lower level values than for low-speed driving.

Based on literature related to the vehicle traffic theory [30,31] it can be concluded that the characteristics of the electric machine fully corresponds to the characteristics of an ideal power source (motor), and the shape of the torque and the power supply corresponds to the optimal torque supply field and the optimum power supply field. Therefore, it can be stated that from the viewpoint of vehicle traction characteristics the PWM controlled electric machine is an ideal source of drive torque for the vehicle.

The properties of an appropriately controlled electric machine allow for maintaining a very simple form of the electric drive design. The electric machine, powered by an electrochemical battery through an inverter is connected to the drive wheels through the final drive and a single gear allowing to reach the maximum speed of the vehicle.

The value of the total ratio at which the electric machine develops its maximum power in relation to the vehicle's maximum speed can be determined by the following relation:

$$i_c = \frac{n_{ME}}{n_{kmax}} = \frac{n_{ME} r_d \pi}{30 v_{max}} \quad (1)$$

where:

$i_c$  – overall mechanical ratio

$n_{ME}$  – electric machine speed corresponding to vehicle speed

$v_{max}$  – maximum vehicle speed,

$r_d$  – dynamic radius of drive wheel.

This structure in generic form is maintained for all electric vehicles presently available on the market. The total ratio value of the electric machine and drive wheels applied in the drive train system of above vehicles ranges from 7 to approximately 10.

One extremely important feature of electric vehicles that enhance their attractiveness and positively contribute to their traction characteristics is their torque overload (current) ability. This particularly applies to the first regulation area where the electric machine torque can be increased as much as three-fold compared to the rated torque required for the electric machine continuous operation. Due to the durability and, above all, the thermal characteristics of the electric machine during overload operation, the electric machine operation time must be limited. In the second regulation area, the overload ability of the electric machine decreases and will disappear near the point of maximum speed. Figure 2 shows the idea of electric machine overload for mechanical characteristics, with the maintenance of limit of the maximum current value.

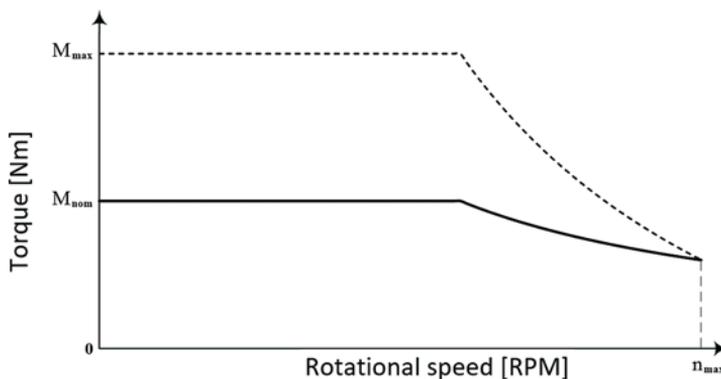


Fig. 2. The idea of electric machine overload

Apart from the extremely favourable traction properties of modern electric machines they are also characterized by high conversion efficiency of electrical energy into mechanical energy and mechanical energy into electrical energy. Thanks to the regeneration mode operation of electric machines it is possible to recover the kinetic energy of the vehicle during regeneration braking. A very good example of this may be a synchronous machine, which, because of its high operating efficiency, is particularly well suited for use in vehicles with an electrical drive. At specific speed and torque ranges, the machine is characterized by notably high efficiency in excess of 85%, taking into account the performance of the control system. However, there are also speed and torque ranges for which the efficiency is much lower. As pointed out above, in electric vehicles the electrical energy from the electrochemical battery is converted into mechanical energy by the traction machine (and vice versa in the case of regenerative braking) with the involvement of the control system. Therefore, it is easy to conclude that the consumption of electricity from the electrochemical battery will in fact heavily depend on the traction machine operating conditions.

Figure 3 shows an example of efficiency distribution characteristics of a PWM controlled electric machine with permanent magnets in relation to its torque and rotational speed.

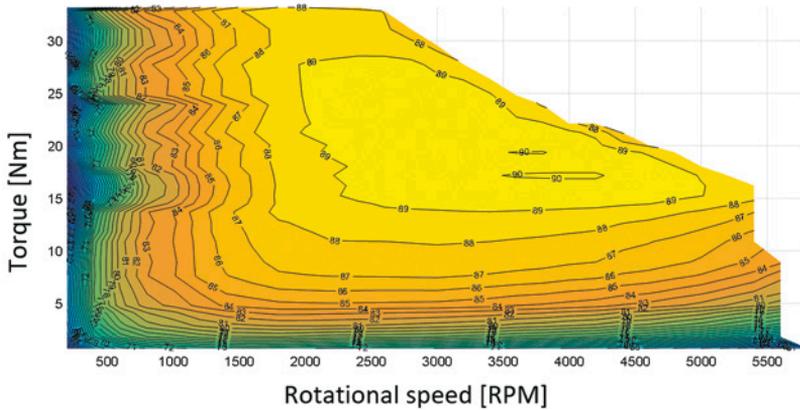


Fig. 3. Example efficiency map of a permanent magnet synchronous machine (PMSM)

It is clear that, taking into account the performance of the electric machine, the load torque range should vary between 50% and 100% of the machine's nominal torque, with the most preferred area of the machine performance being approximately 75% of the nominal torque. It should also be noted that high performance is rather achieved for higher speeds exceeding 50% of the maximum speed. Bear in mind that the highest performance area is approximately 65% of the mentioned maximum speed. In the context of electrical energy to mechanical energy conversion efficiency, it should therefore be expected that operating with the same power at higher speed is more preferable to a higher torque.

Figure 4 shows the electric machine performance characteristics relative to electric vehicle speed, for which the total ratio value has been adjusted for the vehicle to have the ability of achieving a maximum speed of approximately 120 km/h.

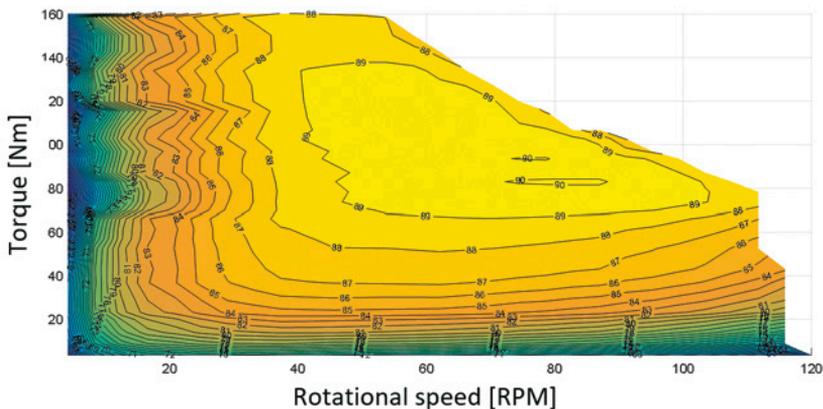


Fig. 4. Electric machine efficiency map relative to vehicle speed with the application of a single constant gear ratio

Thus, it is clear that in the case of an electric drive structure with fixed ratio resulting from maintaining a predetermined maximum vehicle speed, the acceptable efficiency range is exceeding 50 km/h, with the highest efficiency achieved at the travel speed of approximately 85 km/h. If we consider the urban operating conditions, which are characterized by relatively low speed driving and frequent acceleration, it can be found that they force the traction machine operation within the low-value rotational speed range and high-value torque range. This area indicates a lower efficiency value by approximately 5% as compared to the best possible value.

Also the value of the developed torque must be considered, which in the case of acceleration, as well as the cases of increased resistance to the electric vehicle movement, may be insufficient for these factors. As previously mentioned, electric machines are characterized by their temporary current overload ability, which is thus the same for torque. In such case, the electric machine operating time depends on the number of overloads with respect to the rated torque value. It is clear that higher overload current means a shorter operating time of the electric machine. This implies the necessity of close supervision of the electric machine operating states to prevent extensive operating periods in an overload state that may result in damage to the machine. In order to avoid such situations, for an electric drive with fixed ratio it is necessary to dimension the used electric machine to operate for a short time periods with insignificant overload, and even if so, exclusively in extreme load conditions. In the case of electric drives, due to the utilization of energy stored in electrochemical batteries, the operating efficiency of the machine in its overload state is of crucial meaning. It may be assumed that it will be lower than for the area described by the rated torque curve. At a double-overload operation state of the machine the efficiency may decrease as much as by 5% to 8% [32], which is mainly related to the increase of loss in the electrical system of the electric machine and the inverter. Taking into consideration the fact of low efficiency ranges for traction machine operating characteristic for urban operating conditions, the electric machine operating efficiency in relation to the optimal level will decrease by as much as 10%, which undoubtedly will result in increased draw of electrical power from the electrochemical battery pack.

In order to increase the operation efficiency of the traction machine, it is necessary to "shift" its operating points in the direction of optimal efficiency also for lower driving speeds. It was possible to match the characteristics of the traction machine to the maximum speed of the vehicle by adjusting the mechanical ratio. Similarly, however with a different total transmission ratio, it is possible to adjust the characteristics of electric machine to the vehicle speed range characteristic for urban operation conditions. Fig. 5 shows the characteristics of electric machine in relation to the speed range limited by a 50km/h speed value.

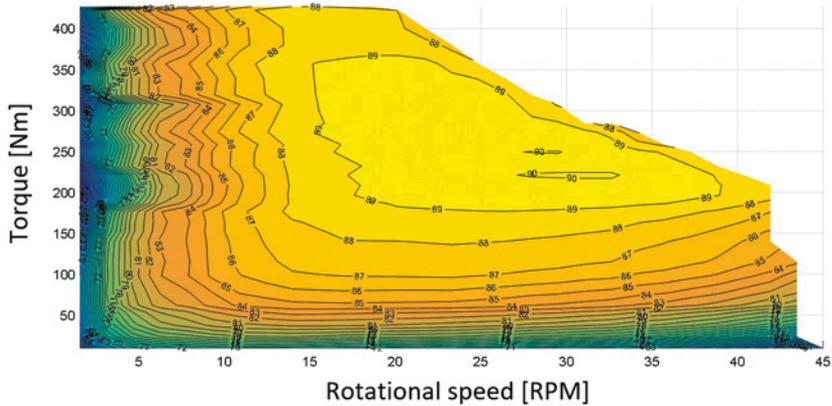


Fig. 5. Electric machine efficiency map relative to vehicle speed with the application of a constant gear ratio with limiting the maximum speed to approx. 50 km/h

The efficiency map shown in Fig. 5 clearly indicates that in relation to the map shown in Fig. 4, the area of the highest operation efficiency of the electric machine is available from a speed of 15 km/h. In addition, the drive train system obtains a very high torque with no necessity of current overload to the electric machine and its control system. Note also that for such specified drive train parameters, the electric machine is operating at a very high efficiency level.

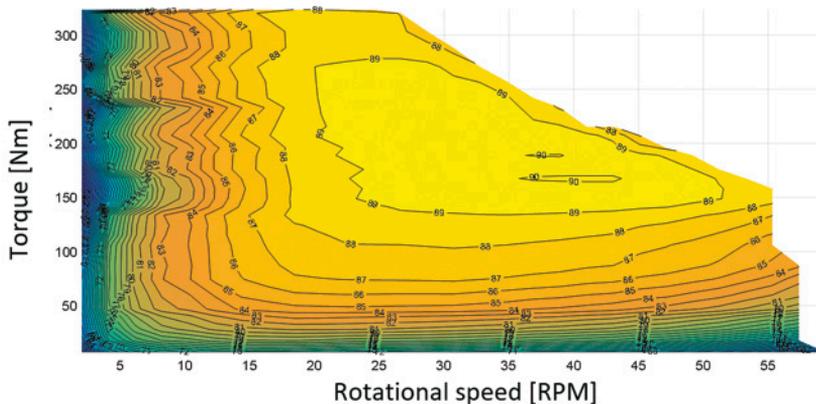


Fig. 6. Electric machine efficiency map relative to vehicle travel speed with the application of a gear ratio allowing high operation efficiency of the electric machine within a speed range of 40 km/h - 60 km/h

If another transmission ratio is matched to adjust the maximum efficiency area to the vehicle speed range of 40 km/h - 60 km/h, it may be stated that virtually the entire speed range of 15 km/h - 100 km/h incorporates the electric machine operating area of high operating efficiency. An example characteristic of electric machine efficiency for this ratio in relation to the electric vehicle speed is shown in Figure 6.

The completed analyses have shown that for the considered example it is sufficient to use three mechanical gear ratios to provide the high operation efficiency of the electric machine within an extremely wide speed range of the electric vehicle. Another advantage of this solution is the possibility of obtaining high torque at the drive train system output without significantly overloading the electric machine. In this case, the use of this property of the PWM controlled electric machine may be limited only to cases with high demands on torque especially at low speed, e.g. when a grade is present. The set of characteristics shown in Figures 5, 6 and 4, i.e. for a transmission ratio allowing the implementation of electric vehicle travel at a maximum speed result in obtaining the characteristic of a maximum torque achievable with the use of an electric machine (whose characteristic is shown in Figure 3), as shown in Figure 7.

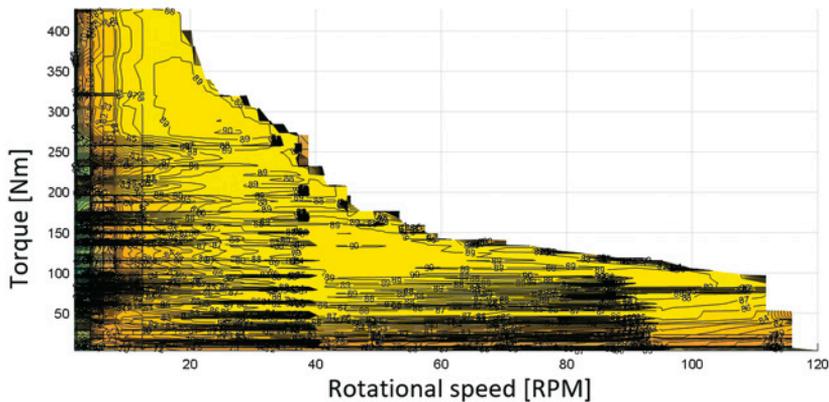


Fig. 7. Electric machine efficiency map relative to vehicle speed with the application of three gear ratios

The value and number of mechanical gear ratios will depend on the vehicle parameters, operating conditions or parameters of the applied electric machine and must be analyzed and matched individually for each case.

To simplify, if the electric machine with permanent magnets is equaled to a corresponding direct current machine, then the electric power drawn from the electrochemical batteries in their steady state can be expressed as follows:

$$\begin{aligned}(U_b I_b) \eta_{CU} &= U_s I_s = c \phi \omega I_s + R I_s^2 - \text{motor operating mode} \\ U_b I_b &= (U_s I_s) \eta_{CU} = c \phi \omega I_s - R I_s^2 - \text{generation operating mode}\end{aligned}\quad (2)$$

The electromagnetic torque of the electric machine is described as follows:

$$c \phi \omega I_s = M_e \quad (3)$$

also, simultaneously, the torque at the electric machine output is related to the following relation:

$$M_e = M_m + M_{strm} \quad (4)$$

If we assume that the torque of mechanical loss is

$$M_{strm} = c_1 + c_2 \omega^2 \quad (5)$$

then the battery power, depending on the operating mode of the drive can ultimately be expressed as:

$$(U_b I_b) \eta_{CU} = U_s I_s = M_m \omega + c_1 \omega + c_2 \omega^3 + R I_s^2 \quad (6)$$

$$U_b I_b = (U_s I_s) \eta_{CU} = M_m \omega - c_1 \omega - c_2 \omega^3 - R I_s^2 \quad (7)$$

where:

$U_b, I_b$  – voltage and current of electrochemical battery;

$U_s, I_s$  – voltage and current of motor;

$\eta_{CU}$  – control system performance;

Accordingly, the battery load during acceleration and at constant speed travel, as well as during braking can be expressed in the following generic form:

$$N_{bp} = \frac{N_m}{\eta_m \eta_{RI}^2 \eta_{CU}} \quad (8)$$

$$N_{bh} = N_m \eta_m \eta_{RI}^2 \eta_{CU} \quad (9)$$

taking into account: the drive train system mechanical efficiency, the copper loss in the windings of the electric machine and the efficiency of the system controlling the electric machine operation.

If the presented equations describing the capacities for electric machine motor and generator operating modes in an electric drive without transmission are applied for a multi-speed transmission with retention of identical speed-torque conditions at the drive train system output, the following will be obtained:

$$N_{bp}^i = \frac{N_m^i}{\eta_i \eta_m^i \eta_{RI}^i \eta_{CU}^i} \quad (10)$$

$$N_{bh}^i = N_m^i \eta_i \eta_m^i \eta_{RI}^i \eta_{CU}^i \quad (11)$$

taking into account the mechanical efficiency of additional mechanical ratio.

Assuming identical speed-torque conditions at the drive train system output for both electric drive configurations, the following equality will be obtained:

$$N_m = N_m^i \quad (12)$$

Because identical drive train system load conditions are assumed, this formula can also be applied with regard to mechanical energy at the drive train system output i.e.

$$E_m = E_m^i$$

Thus, to justify the application of an additional regenerative-speed transmission, the case  $E_{b_p} > E_{b_p}^i$  and  $E_{b_h} > E_{b_h}^i$  must occur, which means that the energy expended at a cycle from the electrochemical battery for the drive train system with additional regenerative-gear transmission must be lower than for a system without this transmission. On the other hand, in the case of energy stored in electrochemical battery as a result of the regenerative braking process, with additional transmission used in the electric drive, the energy must be greater than without it.

This will occur, if:

$$\frac{1}{\eta_m \eta_{RI}^2 \eta_{CU}} > \frac{1}{\eta_i \eta_m^i \eta_{RI}^i \eta_{CU}^i} \text{ and } \eta_m \eta_{RI}^2 \eta_{CU} < \eta_i \eta_m^i \eta_{RI}^i \eta_{CU}^i \quad (13)$$

The above implies that the efficiency of the used regenerative-speed transmission in comparison to electric drives without such transmission under identical load conditions at the driving wheels should remain within the range of:

$$\frac{\eta_m \eta_{RI}^2 \eta_{CU}}{\eta_m^i \eta_{RI}^i \eta_{CU}^i} < \eta_i < 1 \quad (14)$$

The fulfillment of above criterion will ensure the achievement of lower electric power draw from the electrochemical battery and thus the increase of the driving range of electric vehicle with the use of an additional, regenerative-speed mechanical transmission.

### 3. Simulation Comparative Study

The conducted reasoning creates a basis to assume that the application of a greater number of gear ratios for an electric drive, that can be selected depending on the vehicle operating conditions, will result in benefits in terms of reducing the electric power draw from the electrochemical battery.

In order to confirm the validity of this assumption it is best to carry out simulation studies for different electric drive configurations, which will allow the comparison of operating parameters for selected drive train system components, and especially the battery power draw. To establish a benchmark for later comparison in all configurations adopted,

an identical load cycle was adopted, represented by the selected statistical driving cycle and identical vehicle and drive train system parameters, including the electric machine - electrochemical battery pack assembly.

Since the study aims at determining only qualitative and not quantitative parameters for the cases analyzed, in order to facilitate the calculation process, the simplest, however sufficiently accurate, mathematical models of drive train system components have been selected. The study used the specifications of a four-speed transmission system of Fiat Seicento vehicle, as it is most likely that this precise vehicle type will be soon found used as a means of transport for urban areas.

The simulation studies were carried out with the use of specifications presented in Table 1.

**Table 1. Specifications adopted for simulation studies**

Vehicle Data	
Vehicle Weight [kg] (without battery)	950
Rolling Resistance Coefficient $f_r$ [-]	0.01
Aerodynamic Resistance Coefficient $c_x$ [-]	0.45
Frontal Area of the Vehicle $A$ [m <sup>2</sup> ]	1.52
Dynamic Radius of Wheel $r_d$ [m]	0.297
Final Drive Ratio $i_g$ [-]	3.96
Mechanical Gear ratio	
I	3.8
II	2.25
III	1.55
IV	1.1
Electric Machine PM	
Voltage DC [V]	144
Continuous Rated Power [kW]	25
Maximum RPM	5500
Maximum Torque [Nm]	115
Electrochemical Battery Pack	
Voltage [V]	144
Nominal Capacity [Ah]	105

The vehicle and its drive train system were modeled with the use of Matlab Simulink®. To establish the calculation model, the mathematical models for particular electric drive components were used.

### Vehicle Motion Resistance Components

This resistance force can be determined by the following formula:

$$F = F_t + F_w + F_p + F_b \quad (15)$$

where:

$F$  – resistance to motion force;

$F_t$  – rolling resistance force;

$F_w$  – grade resistance force;

$F_p$  – aerodynamic resistance force;

$F_b$  – inertia force.

By limiting to motion in flat road conditions, the grade resistance force may be excluded. Hence, the resistance to vehicle motion can be expressed by the sum of rolling resistance, aerodynamic resistance and inertia forces

The force of rolling resistance depends on the conditions present at the tire to road contact point. This force is determined by the following formula.

$$F_t = mgf \quad (16)$$

where:

$m$  – vehicle weight;

$f$  – rolling resistance coefficient.

The force of aerodynamic resistance can be calculated by the following formula:

$$F_p = 0,047c_xAv^2 \quad (17)$$

where:

$c_x$  – aerodynamic resistance coefficient;

$A$  – frontal area of the vehicle;

$v$  – vehicle velocity in km/h.

The inertia force of is related to the impact of the vehicle weight during acceleration and braking. Its value can be determined by the following formula:

$$F_b = m\delta \frac{dv}{dt} \quad (18)$$

$$\delta = 1 + \frac{I_{me} i_c^2 \eta_T + \sum I_k}{mr_d^2} \quad (19)$$

where:

$\delta$  rotational inertia replacement coefficient;

$i_c$  – overall mechanical ratio in drive train system

$I_{me}$  – inertia of electric machine;

$I_k$  – inertia of vehicle wheels;

$\eta_T$  – efficiency of torque transmission system;

$r_d$  – dynamic radius of drive wheel.

In the case of vehicle motion on a flat road the expression for resistance forces will be as follows:

$$\begin{cases} M_{ME} i_0 \eta_T \frac{1}{r_d} = mgf + 0,047c_x A v^2 + m\delta \frac{dv}{dt} & \text{for driving} \\ M_{ME} i_0 \frac{1}{\eta_T} \frac{1}{r_d} = mgf + 0,047c_x A v^2 - m\delta \frac{dv}{dt} & \text{for braking} \end{cases} \quad (20)$$

### The model of synchronous motor with permanent magnets

Synchronous machine with permanent magnets may be described by the following set of formulas related to the rotor with permanent magnets installed onto it, rotating at a synchronous speed.

$$\begin{cases} \frac{d\psi_d}{dt} - \omega\psi_q + Ri_d = u_d \\ \frac{d\psi_q}{dt} + \omega\psi_d + Ri_q = u_q \end{cases} \quad (21)$$

where:  $\psi_d = L_d i_d + \psi_{fd}$ ;  $\psi_q = L_q i_q$ ;  $\psi_{fd} = M_{df} \Theta_f$  – constant flux of permanent magnets related to axis d;  $M_{df}$  – corresponding mutual inductances;  $\Theta_f$  – permanent magnet magnetic potential;  $L_d$ ,  $L_q$  – inductance of the d-axis and q;  $i_d$ ,  $i_q$  – currents of axis d and q;  $R$  – stator resistance,  $\omega$  – rotor angular velocity.

The electromagnetic torque is expressed by the following formula:

$$M_e = \frac{3}{2} p \left( (L_d i_d + M_{df} \Theta_f) i_q - L_q i_d i_q \right) \quad (22)$$

where:

$p$  – number of pole pairs,

$$P = \frac{3}{2} (u_q i_d + u_d i_q) \quad (23)$$

$$J \frac{d\omega}{dt} = M_e - M_1 \quad (24)$$

$M_e$  – electromagnetic torque of engine,

$M_1$  – external load torque.

The presented mathematical model of synchronous machine allows to determine the electric machine operating parameters with respect to its electrical parameters and the applied control method. Therefore, it can be used for drive analysis to assess both the quantitative and qualitative conditions and operating parameters of electric machine in the analyzed drive. For qualitative assessment of the drive, in particular when the electric machine model has been defined, it is possible to make use of efficiency maps for a similar machine to the one described in Fig. 3, for the purpose of simplifying the mathematical

calculation model and consequently the complete calculation process for drive train parameters. These maps provide a valuable description of energy changes for particular machine type, as they have been determined experimentally by laboratory measurements. They can easily be used to determine the electrical power value required from the electrochemical battery, especially when the drive train system load conditions have been specified. This condition occurs when we use comparative driving cycles.

### Electrochemical Battery Model

The replacement electrical circuit diagram of electrochemical battery is shown in Figure 8.

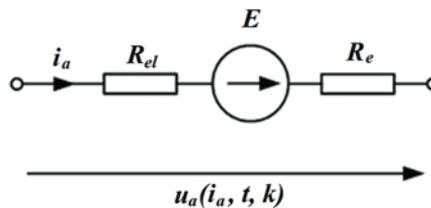


Fig. 8. Replacement diagram of electrochemical battery expressed by electromotive force  $E$  and non-linear resistance of electrodes  $R_c$  and electrolyte  $R_s$  dependent on the capacity of available battery (represented by charge level  $k$ );  $u(i_a, t, k)$  - voltage at battery terminals;  $t$  - time;  $k$  - battery charge coefficient;  $i_a$  - instantaneous battery current ([+] sign for discharge [-] for charging) [33,34]

In accordance with the replacement diagram of electrochemical battery, as shown in Fig. 8, the voltage at the battery terminals may be expressed by the following formula:

$$u(i_a, t, k) = E(i_a, \tau, k) - i_a(t)R_w(i_a, t, \tau, k) \quad (25)$$

where:

$E(i_a, \tau, k)$  - instantaneous value of electromotive force;

$R_w(i_a, t, \tau, k)$  - instantaneous value of non-linear internal resistance;

$i_a(t)$  - instantaneous value of battery current ([+] sign for discharge [-] for charging).

The change of battery charge level represented by coefficient  $k$  can be described by the following relation:

$$k' = k - Q_m^{-1} \int_{i_i}^{i_i+m} \eta_A(i_a, \tau) i_a(t) dt \quad (26)$$

where:

$k$  - initial level of charge

$Q_m$  - capacity of a fully charged electrochemical battery under rated conditions;

$\eta_A(i_a, \tau)$  - utilization coefficient of energy stored in electrochemical battery associated with the presence of Peukert's phenomenon and expressed by the following relation:

$$\eta_A(i_a, \tau) = \left( \frac{i_a(t)}{I_n} \right)^{-\beta(\tau)} \quad (27)$$

When the instantaneous value of current is lower than the rated value the coefficient assumes a value greater than 1. The exponent  $\beta$  depends on the type of electrochemical battery.

To achieve the functionality of the above presented mathematical model of electrochemical battery, the characteristics of the non-linear electromotive force and internal resistance of the battery must be known. These characteristics can be determined based on the time characteristics of the voltage at the battery terminals with the discharge and charging currents of different values, as obtained by conducting laboratory measurements [33, 34].

### Model of Mechanical Multi-Speed Transmission

In the case of mechanical transmission the modeling was conducted exclusively for the energy parameters of the transmission that condition the operating efficiency of the transmission in relation to the gear ratio.

In the case of mechanical pinon gears the efficiency of drive torque transmission can be expressed by the following relation:

$$\eta_i = 1 - k\mu \frac{1}{z_1} (1 - i) \quad (28)$$

where  $k = \frac{8}{\pi \sin^2 2\alpha}$

and

$\mu$  – coefficient of tooth friction;

$\alpha$  – obliquity pressure angle

$z_1$  – number of pinion teeth

$i$  – kinematic ratio of gear

## 4. Study on the Effect of the Electric Drive structure on Electric Energy Consumption

Using the presented mathematical models for the electric drive components, a comparative simulation study of a drive fitted with a constant transmission and a multi-speed transmission was conducted. Because the simulation study was chiefly aimed at determining the energy parameters, at this stage the synchronization process for gear shifting in a multi-speed transmission was omitted, assuming a simplified approach that shifting occurs automatically, under load, without affecting the speed-torque conditions at the drive wheels. This approach implies the assumption of a drive cycle which does not

specify the shifting phase that in turn significantly affects the speed-torque conditions of the vehicle's drive train system.

The comparative studies were carried out for a NEDC driving cycle, shown in Figure 9.

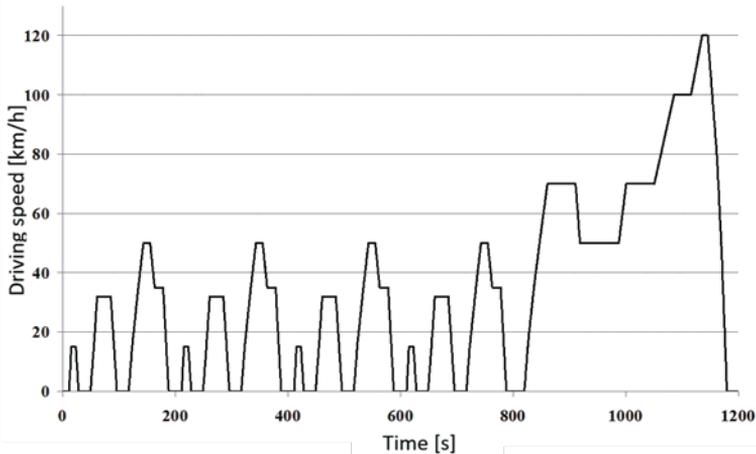


Fig. 9 NEDC driving cycle

The selection of the driving cycle was determined by the fact that it reflects both driving in urban conditions suitable for movement in urban and for non-urban conditions, i.e. when driving on motorways and ring roads at higher speeds.

Figures 10 and 19 presents the results of computer simulations for the individual components of an electric drive using a mechanical multi-speed transmission and for the simultaneous comparison of an electric drive system featuring an electrical machine connected directly to drive wheels via a gear unit of a constant gear ratio. The assumed velocity criterion for the gear ratio shift was defined for the electric machine operation within the speed range in which the highest efficiency area occurs.

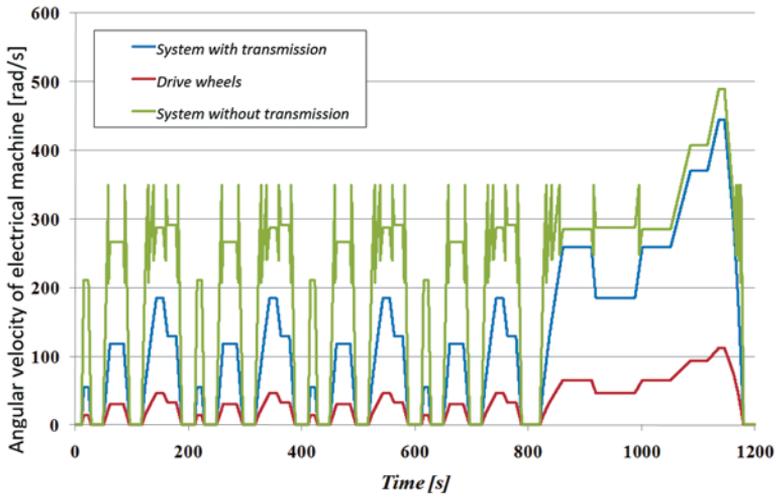


Fig. 10. The comparison of speed process for the drive wheels and the electric machine in an electric drive train system with a multi-speed transmission and without an additional transmission

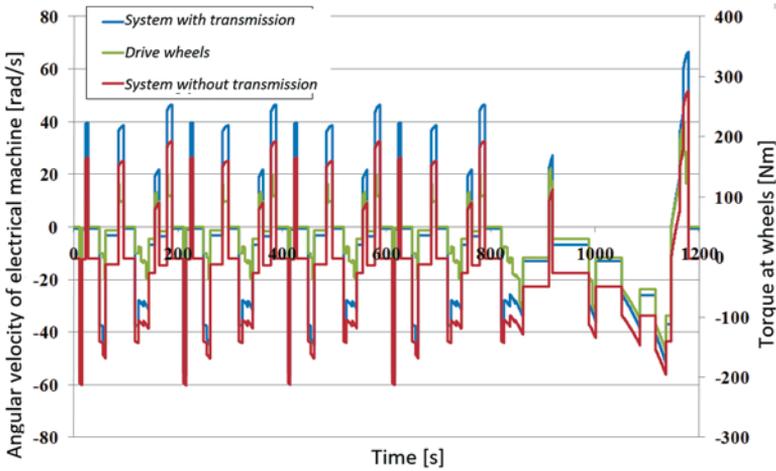


Fig. 11. The comparison of torque process for the drive wheels and the electric machine in an electric drive train system with a multi-speed transmission and without an additional transmission

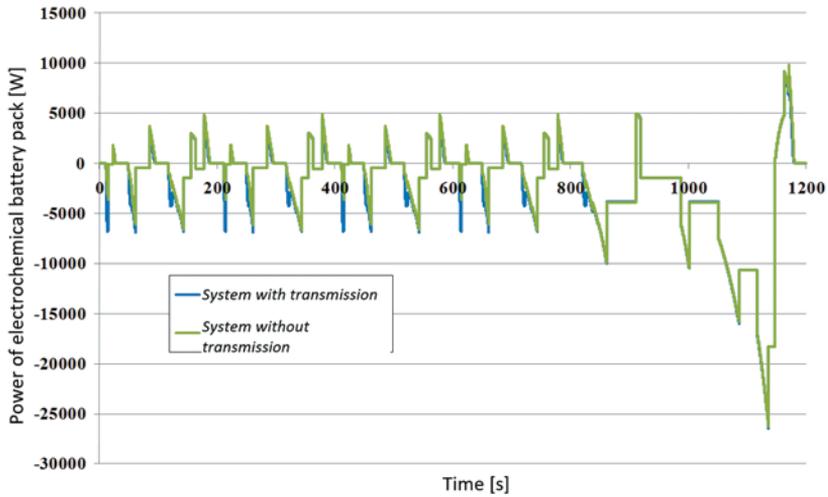


Fig. 12. The comparison of battery power process in an electric drive train system with a multi-speed transmission and without an additional transmission

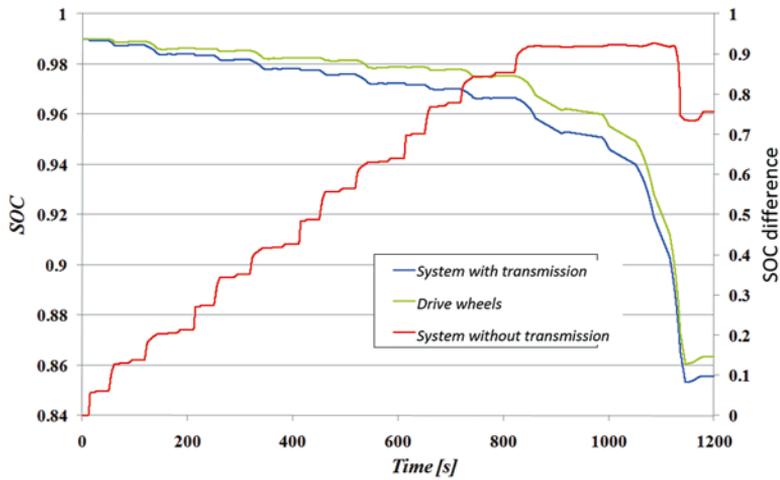


Fig. 13. The state of charge process (SOC) of an electrochemical battery featured in an electric drive train system with a multi-speed transmission and without an additional transmission

As it is visible from the conducted simulation studies, the use of an additional multi-speed mechanical transmission between the electric motor and the drive wheels results in lower discharge levels of the electrochemical battery. This situation will in practice influence the driving range of the vehicle. Figure 14 presents a comparison of driving ranges for the analyzed electric vehicle drive configurations, moving as per the driving cycle shown in Fig. 9. It is clearly visible that the use of an additional multi-speed transmission increases the driving range.

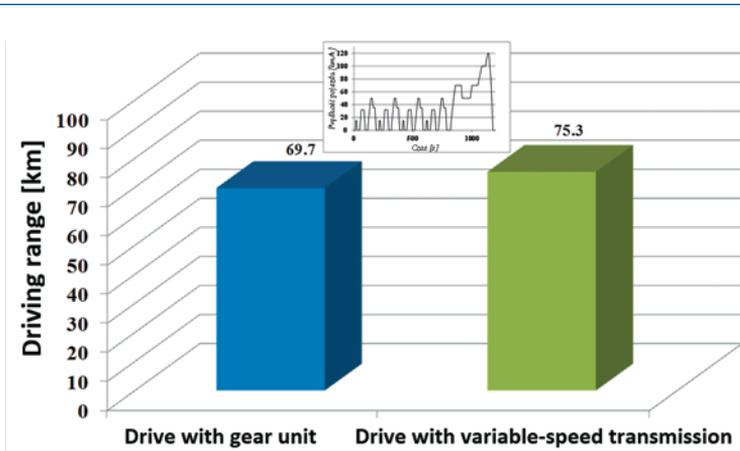


Fig. 14. The comparison of driving range for an electric drive train system vehicle with a multi-speed transmission and without an additional transmission

The increase of the driving range, despite the use of an additional element of the drive train system in the form of multi-speed transmission, is therefore the result of a more favorable distribution of instantaneous efficiencies of the individual drive components as well as the overall efficiency of the drive. Figures 15 to 17 show the instantaneous efficiencies during the drive cycle of Figure 9 for the electric machine – inverter assembly, for the mechanical transmission and the resulting value of instantaneous overall efficiency of the electric drive.

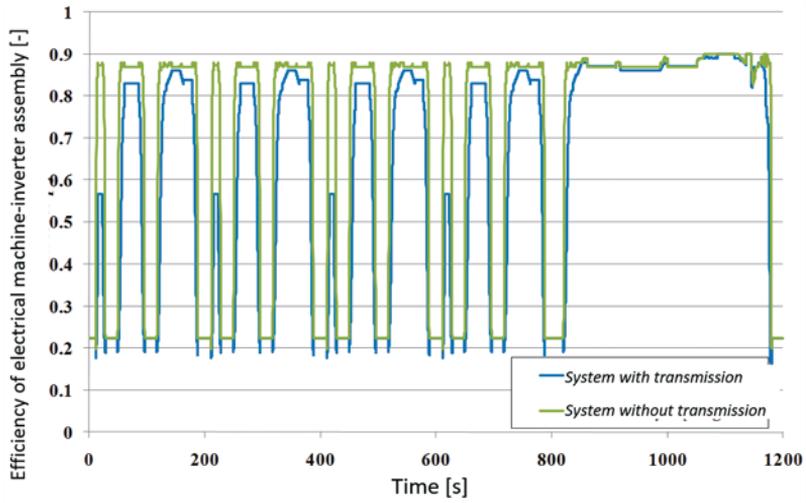


Fig. 15. The electric machine - inverter assembly efficiency

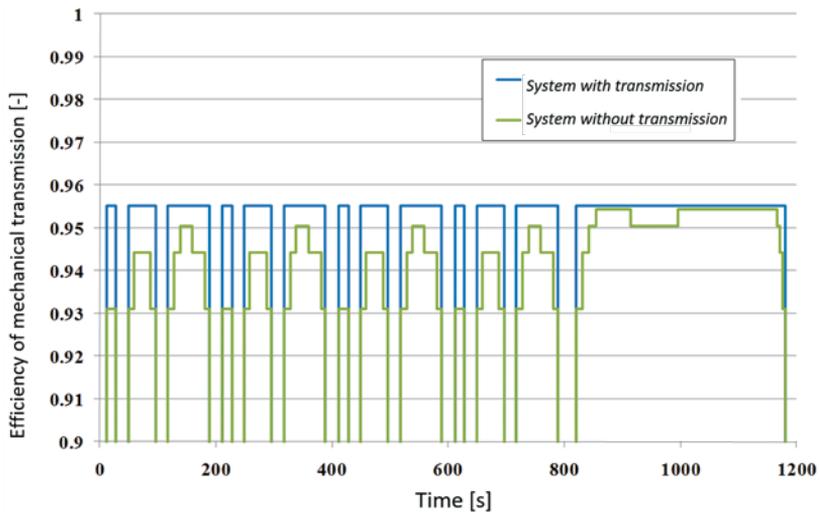


Fig. 16. The mechanical transmission overall efficiency

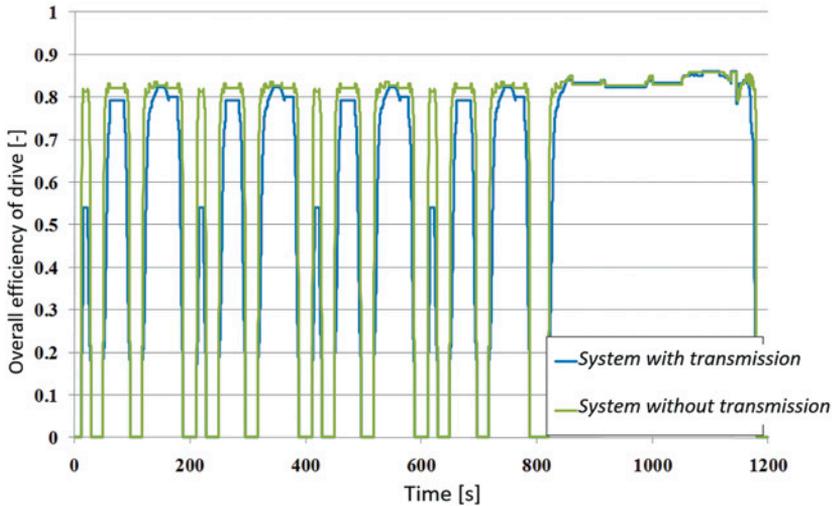


Fig. 17. The electric drive overall efficiency

When comparing the efficiency processes of the traction machine with permanent magnets, taking into consideration the control system efficiency (Fig. 15) and the overall efficiency of the drive (Fig. 17) it is clearly visible that the efficiency is higher for the drive with a multi-speed transmission. Consequently, the driving range for the vehicle with a multi-speed transmission is increased by approximately 8% (see Fig. 14). The obtained value seems insignificant and does not corroborate the advantages of using a multi-speed transmission. The same result can be achieved with an electric drive featuring a gear unit, which is characterized by a much simpler structure and control system, provided that a larger capacity battery is used.

## 5. The Study of the Operation Conditions on the Electrical Energy Consumption Depending on the Electric Drive Configuration

In addition, Figure 13 shows the curve representing the process of the difference in the discharge of electrochemical battery operating in the compared drive train systems under the cycle presented in Figure 9. It is easily visible that the first section indicates a clear increase of the difference that confirms the advantages of applying a multi-speed transmission in an electric drive. However, at the end of the cycle this difference is maintained at a specific, constant level. Finally, at the end of the cycle the difference is reduced. This pattern is directly related to the operating conditions mapped by the driving cycle. This closely corresponds to urban and extra-urban driving cycle sections. One could, therefore, conclude that greater benefits of the use of mechanical transmission with multi gear ratio

are achieved in urban than in extra-urban cycles. In order to check the finding made, the cycle from Fig. 9 was divided into urban and extra-urban driving sections and a simulation study was conducted aiming at specifying the driving range depending on the operating conditions. The cycles corresponding to urban and extra-urban driving are shown in Figures 18 and 19.

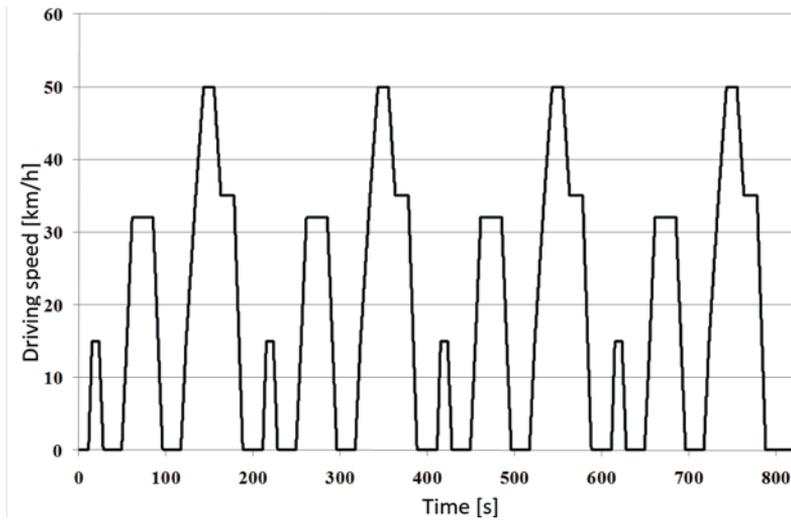


Fig. 18. Urban operating conditions of a vehicle with electric drive

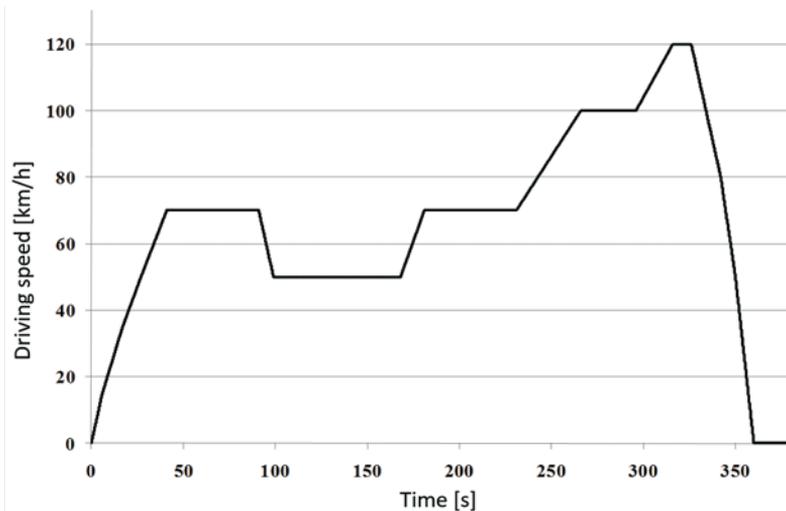


Fig. 19. Extra-urban operating conditions of a vehicle with electric drive

Therefore, it is clearly visible that the cycle shown in Figure 9 reflects two radically different operating conditions of the electric vehicle. In the range of 0 – 800 seconds (see Figure 18), the cycle represents the conditions typical for vehicles traveling in urban areas, and in the range of 800 – 1200 seconds (see Figure 19) we are dealing with conditions similar to vehicle travel outside an urban area. Taking into account the total energy of the cycle, it should be noted that only 11.4% of this energy is used to operate in urban environments, and up 88.6% corresponds to the operation conditions of the vehicle outside the city. Such dominance of extra-urban conditions makes this cycle not sufficiently representative for urban conditions and from the energy consumption point of view it practically corresponds to extra-urban conditions, which are characterized by travelling at constant speed and a very insignificant involvement of regenerative braking. Therefore, it may be stated that both the drive with a multi-speed transmission and also the drive equipped with a gear unit did operate in virtually identical conditions, i.e. at a constant transmission gear ratio (see Fig. 17). This is the reason for such a small difference in the driving range of the vehicle, since only during acceleration and deceleration the drive featuring a multi-speed transmission was characterized by greater efficiency. Therefore, to examine the effects of transmission on energy consumption depending on the operation conditions, simulation studies were carried out for a NEDC split cycle, divided into an urban section (0 to 800 seconds) and an extra-urban section (800 to 1200 seconds).

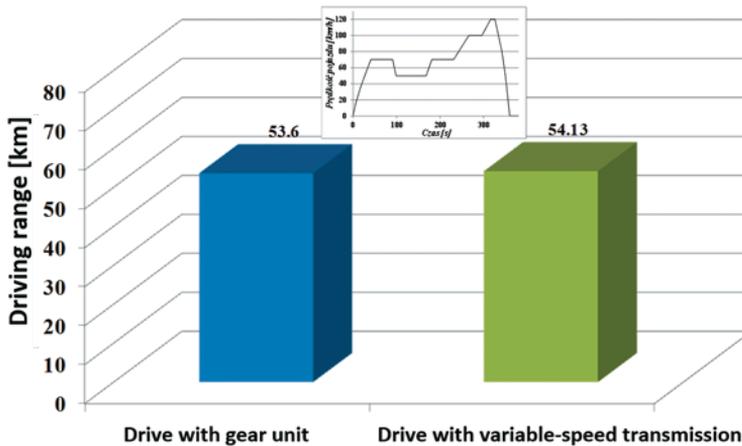


Fig. 20. Driving range for the extra-urban section of the NEDC

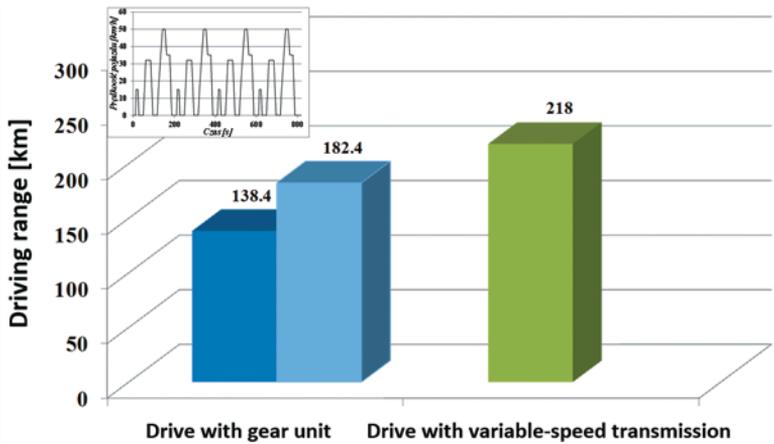


Fig. 21. Driving range for the urban section of the NEDC

The results obtained for the extra-urban section of the NEDC (see Fig. 20) are practically identical for both configurations of electric drive. Based on the results, it is easy to conclude that the use of a multi-speed transmission for such conditions is not justified.

In order to maintain the benchmark, the simulation studies of electric drive for the urban section of NEDC cycle were carried out for identical specifications as in the case of the extra-urban section. Clearly, in this case the use of a multi-speed transmission ensured a driving range higher by as much as 57% in comparison to the drive featuring a gear unit (see Figure 21). Thus, by the use of appropriate mechanical gear ratios the electric power consumption can be nearly halved by limiting the torque, and hence the current of the traction machine traction.

In addition, it has been indicated how important the issue of gear selection is to obtain the lowest possible power consumption for given operation conditions. This is confirmed by the obtained results. The gear unit ratio, which is suitable for driving in extra-urban conditions is unsatisfactory for driving in urban conditions. To improve the energy consumption of such drive in urban conditions it is necessary to select a different transmission ratio value, which will reduce the difference between the multi-speed transmission drive and the gear unit drive to 19.5%. However, in this case the drive drain system with a gear unit adapted to urban conditions cannot be operated in extra-urban conditions. Hence, for an electric drive vehicle to be operated under different conditions it must allow for two transmission ratios at least. This condition is fully met by the multi-speed transmission drive.

The percentage difference between the driving ranges obtained for the electric drive vehicle with a multi-speed transmission and the vehicle with a gear unit operated in different conditions is shown in Fig. 22.

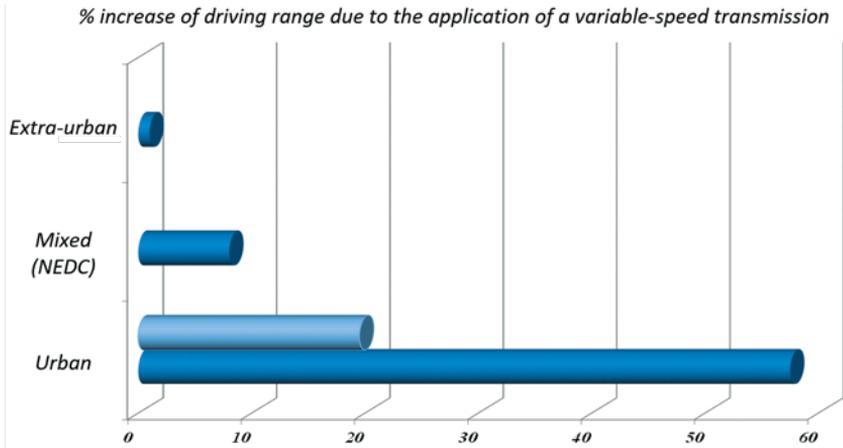


Fig. 22. The impact of the application of a multi-speed transmission on the driving range of an electric vehicle, depending on the operating conditions

It is thus visible that thanks to the use of a multi-speed transmission the electric machine operates at higher efficiency area, which significantly influences the energy consumption and, as a consequence, increases the electric drive vehicle's driving range.

## 6. Conclusions

The analyses carried out indicate that the application of an additional element in an electric drive, being a multi-speed transmission can reduce the electric power consumption. Simultaneously, it is clear that the selection of appropriate mechanical transmission ratio value is crucial due to the increase in driving range, where it is impossible to achieve a satisfactory result with only a single ratio value. It is necessary to use mechanical transmission, which allows to select the transmission ratio value from a set of at least three gears. However, the legitimacy of the multi-speed transmission use for an electric vehicle drive train system is ultimately determined by the operating conditions. The simulation studies carried out clearly indicate that in the case of extra-urban driving the influence of additional mechanical transmission is virtually negligible. In the case of a mixed cycle it can be concluded that it is becoming beneficial. Finally, for urban conditions, featuring an additional multi gear ratio transmission results in a significant increase in the electric vehicle's driving range. The obtained results indicate that it is possible to significantly increase the driving range, whilst using identical electrochemical battery parameters. For vehicles operated exclusively in urban conditions, when increasing the driving range is not the primary objective, the use of a multi-speed transmission allows to limit the energy performance of electrochemical batteries and specifically their rated capacity. Due to the extremely non-linear nature of the electrochemical battery energy performance, in

the case of multi-speed transmission, the most probable expected result will be limiting the rated capacity of the electrochemical battery up to 40%. This will not only reduce the volume and weight of the battery, but also its price. Ultimately, a multi-ratio mechanical transmission should be implemented in this type of (urban) vehicles.

On the basis of computer analyzes conducted it can be stated that the use transmission in electric drive is advisable when the conversion efficiency of electric power is greater for a drive train system featuring transmission in relation to a system without transmission. This is particularly evident for vehicles operated in urban conditions. Firstly, thanks to multi-speed transmission, it is possible to achieve traction machine operation within the highest efficiency range, which increases the overall efficiency of the electric drive. Secondly, the greater number of gear ratios of the multi-speed transmission allows for easier matching of the appropriate ratio to particular operating conditions, thus reducing the energy consumption.

All this gives reason to believe that, despite the excellent traction properties of modern electrical machines used for electric drives, implementing a mechanical transmission allowing to select different gear ratios will significantly improve the operating conditions of the machine.

Continuously Variable transmission (CVT) seems the most suitable for this type of application, due to its wide range of mechanical ratio values available. Another interesting option among the numerous design solutions is the V-belt drive featuring a Kevlar or carbon fiber belt. However, the disadvantages of a practical application of this solution are the relatively high cost and decreased efficiency in comparison to pinion gear transmissions, due to the friction of the belt against the surface the transmission's bevel gears. Additionally, it is necessary to find a solution for the system responsible for setting the required value of mechanical transmission ratio, specifically dedicated for electrical drives, as most state-of-the-art applications feature a hydraulic system to serve this purpose. Another very interesting continuous conversion ratio solution is Electric Variable Transmission (EVT) [37]. This solution consists of an electrical machine integrated with an electromagnetic system resembling a planetary gear unit with two degrees of freedom. The torque is transmitted through magnetic field. Unfortunately, at the moment it is a conceptual solution and seems to be more suited to a hybrid drive with a combustion engine.

The second group are gear transmissions. Among the numerous solutions the Dual-Clutch [38] or planetary gear [39] systems seem to be particularly easy to adapt for electric drives. Their control systems for electric drives will be virtually the same as in the case of combustion drives to which they are dedicated. The number and the values of transmission ratios seem to be an open question. These must be adapted individually, depending on the vehicle parameters and its operating conditions.

Due to their wide application, also manual transmission systems with synchronizers [40, 41, 42] are worth noting. However, their application in electric motors would involve the automation of their operation. Therefore, it is necessary to retrofit the multi-speed transmission with an appropriate control system featuring a suitable actuator to implement

the desired control strategy. Both the control system as well as the control strategy must take into account the phenomena occurring during gear shift. In particular, this is applicable for the synchronization process facilitated by the synchronization unit.

The full text of the article is available in Polish online on the website <http://archiwummotoryzacji.pl>.

Tekst artykułu w polskiej wersji językowej dostępny jest na stronie <http://archiwummotoryzacji.pl>.

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