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## A METHOD OF SYNTHESIZING VEHICLE-DRIVING TESTS BASED ON AN ANALYSIS OF TYPICAL TASKS PERFORMED BY AN INTERNAL COMBUSTION ENGINE

## METODA SYNTEZY TESTÓW JEZDNYCH NA PODSTAWIE ANALIZY TYPOWYCH ZADAŃ REALIZOWANYCH PRZEZ SILNIK SPALINOWY

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### Summary

The article presents an authorial method of synthesizing vehicle-driving tests for determining, by rig testing, performance characteristics of internal combustion (IC) engines in dynamic states representative for the conditions of real engine operation. The method of synthesizing such tests is based on the criteria of similarity between characteristics of the physical quantities that describe the state of engine operation. The similarity criteria apply to the tasks performed by the engine and defined as the consecutive engine operation conditions and the corresponding engine operation states, each representing an elementary action performed by the engine operator by means of the engine. An example synthesis of a test has been presented for the engine speed in the IC engine operation conditions corresponding to the FTP-75 test cycle. The tests built in accordance with the method presented show good repeatability for the empirical data corresponding to various engine speed realizations in the vehicle-driving test.

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**Keywords:** internal combustion engines, vehicle-driving tests, dynamics of internal combustion engines, synthesis of tests

#### Streszczenie

W pracy przedstawiono autorski sposób syntezy testu do badań właściwości użytkowych silników spalinowych w stanach dynamicznych, reprezentatywnych dla warunków rzeczywistego użytkowania silnika. Metoda syntezy testów badawczych jest wykonywana zgodnie z kryteriami podobieństwa charakterystyk wielkości fizycznych opisujących stan pracy silnika. Kryterium podobieństwa dotyczy zadań realizowanych przez silnik, definiowanych jako kolejno występujące warunki pracy i odpowiadające im stany pracy silnika, reprezentujące elementarną czynność realizowaną za pomocą silnika przez jego operatora. Przykład syntezy testu badawczego przedstawiono dla prędkości obrotowej w warunkach pracy silnika spalinowego, odpowiadających testowi jezdnemu FTP-75. Wyznaczone zgodnie z przedstawioną metodą testy wykazują dobrą powtarzalność dla empirycznych danych odpowiadających różnym realizacjom prędkości obrotowej w teście jezdnym.

Słowa kluczowe: silniki spalinowe, testy badawcze, dynamika silników spalinowych, synteza testów

#### 1. Introduction

The performance characteristics of an internal combustion (IC) engine depend on its operation state [4, 6–8, 15]. It is also important that the characteristics are significantly affected by the presence of dynamic states [2–4, 6–8, 10–12, 15, 16]. In the conditions of operation of an automotive engine, the dynamic states predominate; therefore, adequate methods are sought to examine the IC engine performance characteristics in dynamic states simulating real engine operation conditions. There are many publications dedicated to this subject. Results of the examination of pollutant emissions during real operation of an IC engine in a motor vehicle may be found in publications [2, 11, 16]. The operation of IC engines was also examined in non-automotive applications. As an example, results of the examination of pollutant emissions from a marine IC engine during the start procedure and from an engine of a self-propelled machine have been given in [10] and [12], respectively. Works have also been carried out to acquire knowledge of the performance characteristics of IC engines in elementary dynamic states of engine speed and torque, defined by the values of engine speed and torque derivatives with respect to time [4, 8, 15].

The objective of this study is to present the method developed by the authors and intended for the construction of vehicle-driving tests for the examination of IC engines on a test rig in the conditions simulating real operation of the engines. These conditions are determined, first of all, by the vehicle speed process. Hence, vehicle-driving tests, representing real vehicle driving conditions on a chassis dynamometer, are used in type-approval procedures for the examination of pollutant emissions and fuel consumption of automotive engines. The vehicle speed process determines the engine operation state (4, 6–8, 15], first of all the engine speed and torque. Therefore, the tests used for the examination of IC engines on engine test rigs are defined as engine speed and torque vs. time curves [7, 17, 18]. In general, tests for the examination of motor vehicles are built to comply with the following [6, 7]:

- predefined time history of vehicle speed relative to the maximum vehicle speed;
- requirement of faithful simulation of the time history of vehicle speed;
- criteria of similarity between the characteristics of vehicle speed in various domains,
  e.g. time, independent variable in the integral transformation of the time history or values of the process, etc.

When an IC engine is to be examined on a chassis dynamometer, the tests may be built in compliance with the same rules related to engine speed and torque [6, 7]. This study presents a method of synthesizing vehicle-driving tests in accordance with the criteria of similarity between characteristics of the physical quantities that describe the state of engine operation [6, 7]. Every similarity criterion applies to the tasks performed by the engine and defined as the consecutive engine operation conditions and the corresponding engine operation states, each representing an elementary action performed by the engine operator by means of the engine.

Two basic issues may be distinguished in the concept of this work:

- defining the most typical tasks performed by the engine under consideration during its operation in mobile applications: this is the task to analyse the engine operation states;
- synthesizing new tests based on the selected tasks performed by the engine.

In the example presented herein, the IC engine operation conditions were adopted as being determined by the vehicle speed process defined for the FTP-75 test cycle [18]. In the calculations, results of the empirical tests carried out to prepare the FTP-75 test procedure [15] were used. Of course, any other conditions may also be assumed, in the form of vehicle speed vs. time curves or other vehicle-driving tests, whether used for type-approval [18] or special [5] purposes.

### 2. Tasks performed by the engine

The task consists of consecutive engine operation conditions and the corresponding engine operation states. It is assumed that the task performed by the engine is to correspond to the attaining of a simple target, such as e.g. raising the engine crankshaft speed from an initial to a target value at a predefined load or maintaining a preset crankshaft speed at a constant or varying load. Thus, the task performed by the engine is a process of transition along a "relatively short route" from initial to target engine operation conditions.

A key issue in this definition is the notion of "relatively short route" of the transition. It has the meaning that the course of changes in all the physical quantities during the execution of the task may be geometrically represented by straight lines (forcing literally "the shortest route", in geometrical sense) of by differentiable curves with low curvature, when the realization of a specific process "along a straight line" (in geometrical sense) is physically impracticable. If the functions representing the physical quantities that describe the engine operation conditions had distinct differentiability limits, the process under consideration should be divided into more tasks, because this would mean that a simple transition from initial to target conditions does not take place.

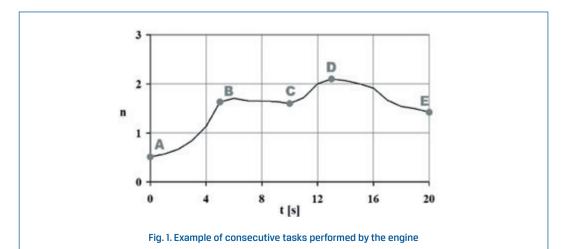
An example of the tasks performed by the engine has been presented in fig. 1 in the form of an IC engine speed vs. time curve, being a fragment of the time history of the relative engine speed prescribed in the FTP-75 procedure.

For the process x(t) under consideration, the relative process value  $x_w(t)$  is defined as the ratio of process value x(t) to standard deviation  $\delta[x(t)]$  of the said process:

$$x_{w}(t) = \frac{x(t)}{\delta[x(t)]}$$
 (1)

In the case of real time curves, as opposed to mathematical functions, the following two factors should be formally taken into account at the estimation of differentiability limits. The first one is the frequency characteristic of a specific time curve [14], especially its upper frequency limit. The other one is the technical inaccuracy of the analyses, considered as acceptable. The work described herein does not include such analyses because this article is only to present the method proposed rather than to carry out an actual synthesis of a real test. Therefore, some judgments as regards the categorizing of time curves as single tasks were only made intuitionally.

The transition from point A to point B is a single task, although it is not realized "along a straight line". Similarly, the transitions between consecutive points are characterized by "minor irregularities" in the curve, but the irregularities arise from the specific nature of engine operation and not from a change in the target adopted. On the other hand, the transition from point A to point E must not be treated as a single task, because the nature of changes in the engine speed varies during that period.



Every task performed by the engine consists of individual engine operation parameters. Nevertheless, the parameters taken as a whole may also be treated as preset engine operation conditions.

Within the conditions defined by a task, individual engine operation states may also be analysed (as realizations of an engine operation process); as an alternative, an average state (in a specific sense) taking place in the task may be determined.

Therefore, the task considered as preset engine operation conditions and the state of engine operation in specific conditions will be dealt with in the further part of this study.

# 3. Analysis of the vehicle-driving test with respect to the tasks performed by the engine

Since the notion of a task performed by an engine has been defined as a process of transition along the shortest route from initial to target engine operation conditions, then the task represents an elementary action performed by the engine operator by means of the engine. Of course, an important problem now arises as regards the isolation of the tasks thus defined from the actual engine operation process.

A task performed by an engine, as the notion describing engine operation conditions, may be defined in the domain of any physical quantities having an impact on the engine operation.

In the calculations presented as an example, an assumption was made that the parameter used for describing the engine operation would be engine speed. This is because of the fact that the other quantities, such as the braking torque applied as a load to the engine crankshaft or the parameter that describes the engine steering, are closely related to the engine speed. As an example, the braking torque is determined by the resistance to vehicle motion and this resistance is closely correlated with the engine speed.

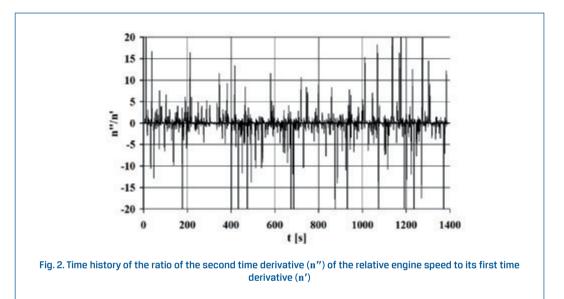
It should be stressed, however, that such a decision is only an example of the possible options, which helped to simplify the presentation of the method proposed. Of course, other physical quantities, if necessary, may also be taken into account when isolating individual tasks performed by the engine.

A problem of considerable importance, from not only theoretical but also practical point of view, is the method of selecting the inter-task boundaries. In the situation that the engine operation conditions (and, in consequence, the tasks performed by the engine) are defined by engine speed alone, the boundaries will be the points considered landmarks in the engine speed vs. time curve.

A bare variation in the engine speed should not be considered an inter-task boundary criterion, as the engine speed may change within a single task. When the variation in a physical quantity under consideration is assessed, an analysis of time derivatives of the said quantity is advisable. In authors' opinion, the ratio of the second time derivative of the physical quantity under analysis to the first time derivative of this quantity should be adopted as a criterion of determining the inter-task boundaries. This ratio provides information about the relative variation in the first derivative of the physical quantity under analysis with respect to time.

However, there is a problem with determining the threshold value of such a ratio. In practice, this should be a value arbitrarily assumed by the researcher on the grounds of his/her knowledge and experience.

A time history of the ratio of the second time derivative of the relative engine speed to its first time derivative has been shown in fig. 2.



Based on the example empirical data obtained from a realization of the FTP-75 test, the threshold value of the ratio between the time derivatives of the relative engine speed has been arbitrarily chosen as being at a level of 0.8; in consequence, boundaries between consecutive tasks performed by the engine were assumed to be situated wherever the curve representing the said ratio of the second time derivative of the relative engine speed to its first time derivative exceeded this threshold value.

Of course, there is also a problem regarding the optimization of the threshold value that defines boundaries between consecutive tasks. If the threshold value were too high, there would be a small number of tasks and the changes in the physical quantity under analysis might be bigger than they would be if too low a threshold value were adopted. In the latter case, in turn, too many tasks would be isolated and they would sometimes be of a similar nature.

Fig. 3 shows a time history of the relative engine speed, with the inter-task boundary points having been marked as black dots. Thus, the grey lines linking the black dots represent individual single tasks performed by the engine.

The problem of sensitivity of the method to the threshold value adopted is, however, far less conspicuous if the ratio of the second time derivative of the physical quantity under consideration to the first time derivative of this quantity is analysed, in comparison with the situation where only the second derivative of this quantity with respect to time is analysed.

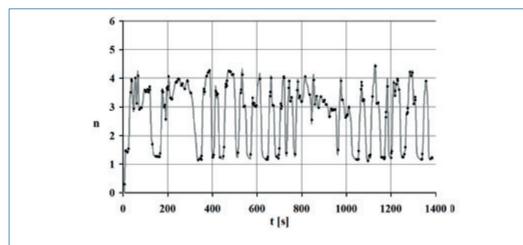


Fig. 3. Time history of the relative engine speed (n), with the inter-task boundary points having been marked as black dots

In the case of discrete time histories of the physical quantity under analysis, some of the tasks having been isolated may happen to be very short, or may even consist of two extreme points. Such a case clearly shows that the use of the method proposed requires the frequency characteristic of the physical quantity under analysis to be taken into account.

Fig. 4 presents a time history of engine speed, where the tasks consisting of at least 8 points have been marked out. At a sampling frequency of 1 Hz, this means that such tasks would be extended in time for at least 7 s. The boundary points of measuring the relative engine speed have been marked as black dots; the grey dots represent the measuring points that have been classified within individual tasks.

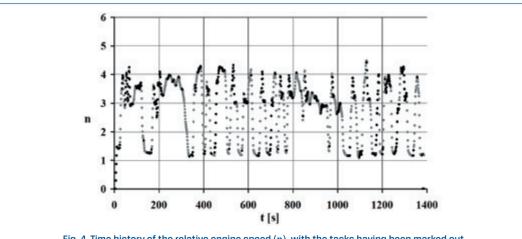


Fig. 4. Time history of the relative engine speed (n), with the tasks having been marked out

The question how long tasks should be taken into account when analysing the test results is quite problematic and the answer depends on the objective of using the method proposed. For the lucidity of presentation of the calculation results, the tasks consisting of at least 8 points were selected. Thanks to this, only somewhat more than 50 tasks were chosen and the results obtained made it possible to demonstrate the method in a clear way.

It should also be mentioned how the necessary procedure would have to be changed if time histories of physical quantities other than the engine speed were used for describing the tasks performed by the engine. In such a case, a procedure of determining the threshold value should be separately carried out for each of the physical quantities involved and then the inter-task boundaries should be identified wherever a boundary point would be present within at least one of the time curves under consideration. For the boundaries thus determined, the above procedure of choosing the tasks that consist of the predefined number of points may be continued.

As it can be seen, the recommended procedure will remain almost identical if a higher number of physical quantities are taken into account in the analysis. However, the length of the tasks identified would change. The more factors are involved that would introduce divisions between individual tasks, the shorter the resulting tasks are. This is why only one parameter describing the engine operation conditions (and, therefore, the tasks performed by the engine) has been used in his study in order to maintain the lucidity of the example analysis results.

# 4. Synthesis of the vehicle-driving test with respect to the tasks performed by the engine

An analysis of the IC engine operation conditions makes it possible to isolate individual tasks performed by the engine. Based on the tasks having been isolated, an appropriate vehicle-driving test can be synthesized.

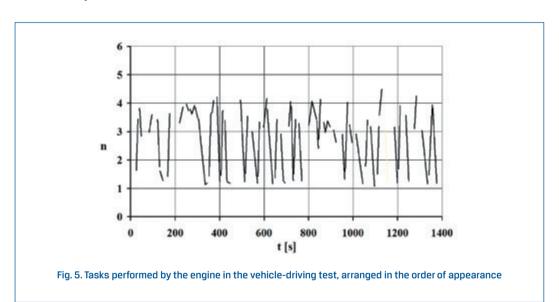
Each of the separated tasks performed by the engine is a process described by selected physical quantities. In the deliberations above, the task was defined on the grounds of relative engine speed alone and it defined the engine operation conditions. On the other hand, these conditions may also be assigned other physical quantities describing the engine operation state, such as e.g. the intensity of emission of selected exhaust gas components. Thus, the engine operation state that constitutes a single task is a set of consecutive values of the relative engine speed and the intensity of emission of selected exhaust gas components.

Every task like this, performed by the engine, may be described parametrically; thus, an approximation of the task will be obtained [1]. Since the task is a process of transition along the shortest route from the initial engine operation conditions, i.e. initial relative engine speed, to the target engine operation conditions, i.e. target relative engine speed, it may be described with the use of the initial and target values of the relative engine speed and approximated by a linear function [1]. Naturally, every task is extended in time; therefore, the task duration time should be additionally taken into account. Moreover, every task may

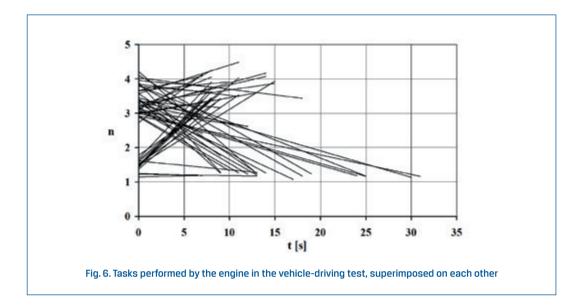
be assigned an effect of engine operation in such conditions, e.g. the emission of selected exhaust gas components.

Finally, each of the tasks performed by the engine within the vehicle-driving test will be described, in the examples presented, by the time of start and end of the task being performed within the driving test (or the task duration time), initial and target relative engine speed, and emission of exhaust gas components.

Fig. 5 shows all the tasks performed by the engine and isolated from the FTP-75 procedure, after having been approximated in a way as described above. The tasks have been arranged in time in the order of appearance in the vehicle-driving test, i.e. in the order identical to their actual succession during the test. The empty spaces between individual tasks reflect the fact that some of the measuring points in the test have not been classified within any of the said tasks.



On the graph presented in fig. 5, the tasks have been described by the task start and end time. If these two figures were replaced with bare information about the task duration time, then all the tasks shown on the graph would be superimposed on each other. Such a situation has been shown in fig. 6.



Thus, all the possible time histories of the relative engine speed, representing the tasks performed by the engine in the vehicle-driving test under consideration, were plotted in the same place. Of course, such a graphical presentation is relatively hard to read because of high density of the curves representing tasks that considerably differ from each other. Therefore, an attempt was subsequently made to divide the existing tasks into groups.

Based on the isolated tasks performed by the engine, several task groups may be distinguished where the tasks would be similar to each other. Visually, this corresponds to the presence of lines in the graph whose slope angles would fall within certain ranges arbitrarily adopted.

Figs. 7–12 show specific task groups, hereafter referred to as types of tasks performed by the engine. Altogether, six types of this kind were discerned:

- type 1, characterized by a growth in the relative engine speed from low to medium;
- type 2, characterized by a growth in the relative engine speed from medium to high;
- type 3, characterized by a drop in the relative engine speed from high to medium;
- type 4, characterized by a slow decline in the relative engine speed from medium to low;
- type 5, characterized by a fast drop in the relative engine speed from medium to low;
- type 6, characterized by the relative engine speed being kept at an almost constant level.

The definitely imprecise terms used in the above classification result from the arbitrary selection of the criteria adopted.

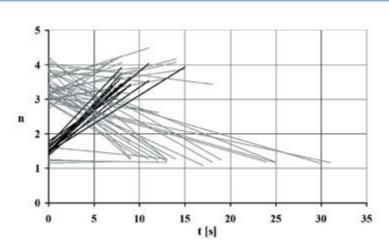


Fig. 7. Type 1 of the tasks performed by the engine

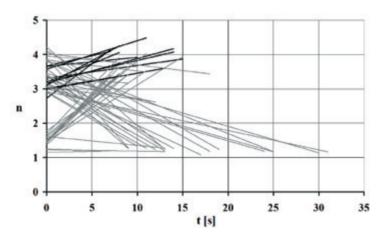


Fig. 8. Type 2 of the tasks performed by the engine

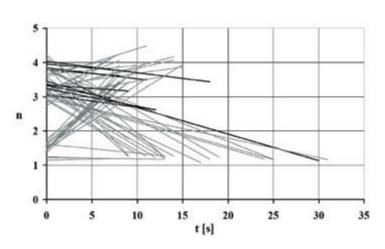


Fig. 9. Type 3 of the tasks performed by the engine

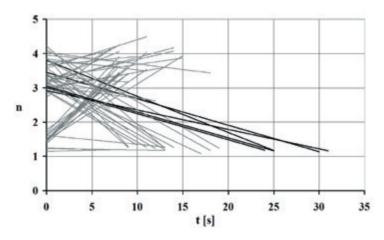
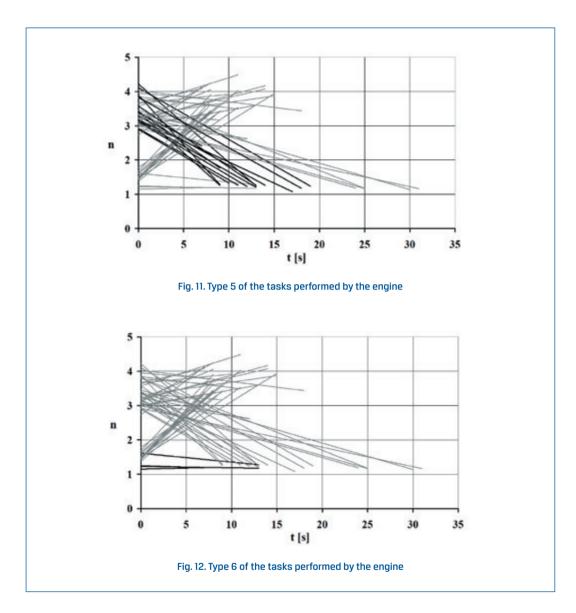
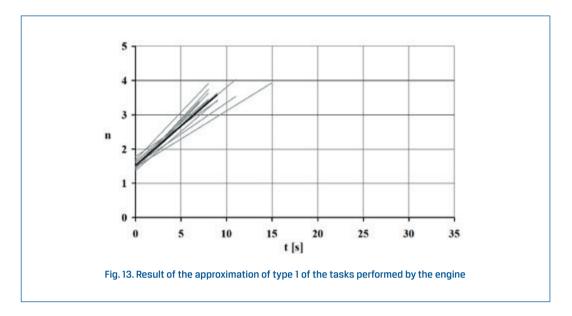


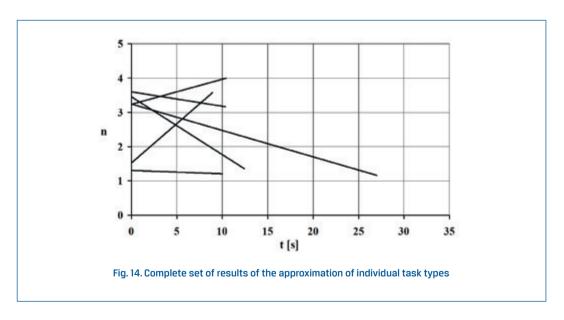
Fig. 10. Type 4 of the tasks performed by the engine



Each of the isolated groups (types) of the tasks may be approximated by a single average task. An example approximation result has been presented in fig. 13. The approximation was carried out according to the least squares method [1].



Results of the approximation of all the six types of the tasks performed by the engine have been brought together in fig. 14.



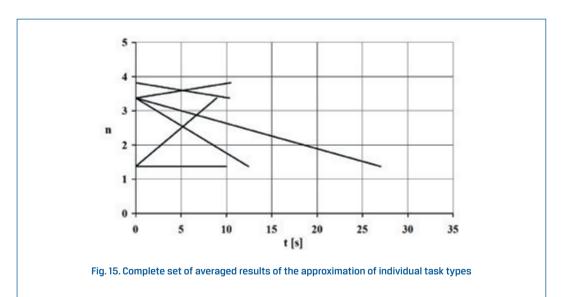
It can be seen in the graph presented in fig. 14 that individual tasks (approximations of individual task types) significantly differ from each other: individual task types both begin and end in different values of the relative engine speed.

To classify the tasks by relative engine speed value, specific values may be adopted as classification criteria, e.g. three main values (low, medium, and high) of the relative engine speed. In this study, the main values of the relative engine speed were adopted as weighted average values of the relative engine speed occurring in the specific tasks. The weights used for calculating the weighted average values represented, in percentage terms, the numbers of occurrences of tasks of a specific type in the vehicle-driving test under consideration.

Each of the said main values of the relative engine speed only occurred in some of the task types.

- the low value of the relative engine speed occurred in tasks of type 1, 4, 5, and 6 (twice at the beginning and twice at the end of the task);
- the medium value of the relative engine speed occurred in tasks of type 1, 2, 3, 4, and
  5:
- the high value of the relative engine speed occurred in tasks of type 2 and 3.

Fig. 15 shows again a complete set of the six task types, but with the main values of the relative engine speed having been averaged.



A difficult issue is the selection of the frequency of appearance of individual task types in the vehicle-driving test being synthesized. Of course, information about the numbers of occurrences of individual task types in the test under analysis would help; nevertheless, an unequivocal solution of this problem cannot be thus found.

The shares of individual task types in the FTP-75 test have been specified in table 1.

Table 1. Shares of individual task types in the FTP-75 test

Task type number	1	2	3	4	5	6
Share in the FTP-75 test	0.246	0.193	0.158	0.088	0.246	0.070

Noteworthy is the fact that the share of tasks of type 1 (characterized by a growth in the relative engine speed from low to medium) is not equal to the sum of the shares of tasks of types 4 and 5 (characterized by a decrease in the relative engine speed from medium to low). Similarly, the shares of tasks of types 2 and 3 are not equal to each other. Although the differences are not too big, this means that if the existing proportions of task types are uncritically maintained in the vehicle-driving test then the task types with increasing relative engine speed would have to occur more often than the task types with decreasing relative engine speed (or vice versa), which is impracticable in real tests.

To solve this problem, one should first pay attention to the reason for such a situation. This arises from the fact that not all fragments of the vehicle-driving test were categorized by the algorithm of isolating individual tasks among the tasks performed by the engine. In consequence, the proportions of shares of individual task types might have been somewhat disturbed. Nevertheless, the balance of the phases of increase and decrease in the relative engine speed must be zero. Thus, the shares of individual task types should be rounded so that the sum of the tasks with increasing relative engine speed is equal to the sum of the tasks where the relative engine speed decreased.

The numbers of occurrences of individual task types, adopted in the example under consideration, have been specified in table 2.

Table 2. Adjusted shares of individual task types in the FTP-75 test - variant 1

Task type number	1	2	3	4	5	6
Number of occurrences in the test synthesized	2	1	1	1	1	1
Share in the test synthesized	0.286	0.143	0.143	0.143	0.143	0.143
Share in the FTP-75 test	0.246	0.193	0.158	0.088	0.246	0.070

The above figures constitute quite a coarse simplification of the numbers of individual tasks in the test being synthesized, aimed at shortening the time of the entire test. Of course, if a more accurate representation of the frequency of appearance of individual task types were desirable, the total number of their occurrences should be increased, with the total time of the vehicle-driving test being thus extended. Nevertheless, this really might be done. A somewhat more developed example of the occurrences of individual task types in the test being synthesized has been presented in table 3.

Task type number	1	2	3	4	5	6
Number of occurrences in the test synthesized	4	2	2	1	3	1
Share in the test synthesized	0.308	0.154	0.154	0.077	0.231	0.077
Share in the FTP-75 test	0.246	0.193	0.158	0.088	0.246	0.070

Table 3. Adjusted shares of individual task types in the FTP-75 test - variant 2

Individual tasks in the vehicle-driving test being synthesized may be almost freely arranged in any order. The only requirement to be met is that each of the successive tests should begin from a relative engine speed identical to the final speed of the preceding task. This requirement can be easily met thanks to the averaging of the initial and target values of the relative engine speed.

A synthesized vehicle-driving test taken as an example and based on the data given in table 2 has been presented in fig. 16.

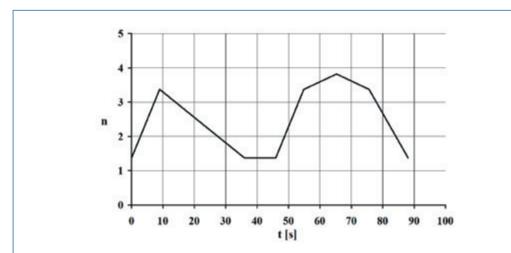


Fig. 16. Example vehicle-driving test synthesized on the grounds of a set of tasks performed by the engine, previously isolated within this work

In the example presented above, all the tasks performed by the engine, isolated from a vehicle-driving test having been carried out, have been used. However, it is also possible to pick only the tasks that meet specific criteria, e.g. are characterized by the highest emission of selected exhaust gas components.

As an example that illustrates the possibility mentioned above, the relative sums of the emissions of exhaust gas components under examination (carbon monoxide, hydrocarbons, and nitrogen oxides) for each task type have been brought together in table 4.

Table 4. Relative sums of the emissions of exhaust gas components under examination, determined for each task type in the vehicle-driving test

Task type number	1	2	3	4	5	6
Total emission	85	61	45	27	16	9

The highest relative sum of pollutant emissions was recorded for the task where the engine speed was rapidly raised; a somewhat lower emission was observed when the engine speed was raised at a lower rate; even lower total emissions could be seen when the engine speed was slowly decreased, and so on. So, the findings presented are consistent with theoretical expectations. The lowest relative sum of pollutant emissions occurred in the task where the engine speed was maintained at an almost constant low level, which is natural, too.

To examine the repeatability of the process of synthesizing vehicle-driving test procedures from independent results of empirical tests, a comparison was made between tests synthesized from four time histories of engine speed recorded during FTP-75 vehicle-driving tests, treated as individual realizations of a stochastic process [13]; the comparison has been presented in fig. 17.

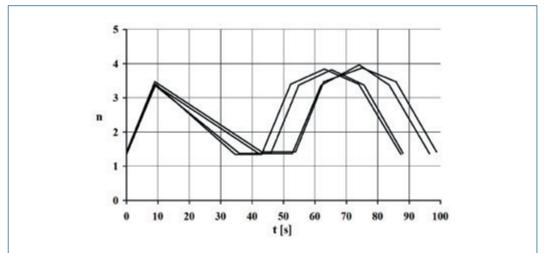


Fig. 17. Vehicle-driving tests synthesized from four realizations of the engine speed process in tests carried out in accordance with the FTP-75 procedure

In spite of considerable differences in the parameters of the vehicle-driving tests having been synthesized, the repeatability of the tests can be clearly seen.

#### 5. Conclusions

The method presented enables the synthesizing of vehicle-driving tests from an analysis of typical tasks performed by an internal combustion engine during tests carried out in accordance with the existing standard vehicle-driving test procedures. The following applications of this method are possible.

- 1. The method presented offers a possibility of simplifying vehicle-driving test procedures prepared in the form of time histories of physical quantities recorded during real drives of a motor vehicle propelled by an internal combustion engine designed for mobile applications. When the motor vehicle is driven in real road traffic conditions, the vehicle operator (driver) performs many various tasks by means of the engine. Thanks to the method proposed to analyse road test results, the said tasks may be isolated and systematized and a vehicle-driving test (to be carried out on an engine test bed or a chassis dynamometer), which would represent the engine tasks recorded during real vehicle drives, may be synthesized.
- The said method makes it also possible to select the tasks performed by an engine during a vehicle-driving test (in the conditions of real road traffic or in laboratory conditions) that are considered most interesting (e.g. from the ecological point of view, as being characterized by the highest emissions of selected pollutants).

The full text of the article is available in Polish online on the website http://archiwummotoryzacji.pl.

Tekst artykułu w polskiej wersji językowej dostępny jest na stronie http://archiwummotoryzacji.pl.

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