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A METHOD OF DISCRETISING THE BELT IN A FLAT MODEL OF A BELT TRANSMISSION

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Summary

The article presents a method of discretising a belt used in a transmission model with any number of pulleys and tensioning rollers and any direction of wrapping the pulleys or rollers. The positions of the discretising points are formulated in a global coordinate system. The method allows for arbitrary placement of pulleys and tensioning rollers. It also consequently allows to calculate the length of the belt, resulting from the geometry of the transmission, and the length of the belt, resulting from the coordinates of the discretising points that lie on its circumference. The method presented here allows to estimate the number of points that would provide satisfactory accuracy of the belt's curvature surrounding the individual pulleys and rollers. In the paper presented comparison between two proposed ways of estimating. Designated points can be used as input data to analyse the belt dynamics with the belt modelled as rigid elements connected to one another by translational or rotational spring-damping elements.

Keywords: poly-V belt, belt transmission, discrete model of the belt

1. Introduction

The belt models that are available in the literature can be divided into continuous models [1, 5, 7, 12] when the belt is considered a stretched string that does not take into account rotational rigidity and discrete [2, 6, 9, 10], most commonly assumed as rigid beam elements joined together by translational, sometimes with additionally torsional, spring-damping elements. The application of a suitable model primarily depends on the assumptions made, in particular on what phenomena will be taken into account in the model. It is particularly problematic to develop appropriate assumptions for an analysis of the model's dynamics. The use of a discrete model facilitates investigating locally occurring

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phenomena in a certain portion of the belt, it is particularly helpful when unusual friction and contact models between the pulley and the belt are used.

It is worth mention that in most of papers presenting models of belts are two-pulley systems, and positions of the elements are treated as already known input data. Simultaneously, there are few works presenting multipulley transmissions, such as [1, 3, 11].

The positions of discrete points are mostly presented in global coordinate system (among others in work [2]), which can be helpful during deriving equations of motion. This way was also included in this work.

The amount of accepted elements into which the belt will be divided is fundamental when adopting the appropriate discrete belt model. This factor and the number of degrees of freedom of individual elements have an influence on the total number of differential equations of motion and, consequently, on the numerical efficiency of the model being developed. On the other hand, the assumption of a small number of elements leads to poor accuracy of mapping the belt deformation, particularly in fragments lying on rolls or pulleys with a small diameter. This also reduces the length of the belt after its discretisation, relative to its actual length. It is worth paying particular attention to the problem of determining the normal force of distinct elements, i.e. both its direction and, perhaps even more problematic, its value. In some models the force depends on the depth of the "penetration" of the two bodies, which in the case of rarely spaced points on the circumference of a wrapped pulley can lead to unrealistic values of this force. An example of such a model is Kelvin-Voigt's [13] contact model.

Improper values of normal force influence bad calculations of values of friction forces. It can consequently cause: large slip between the belt and pulley or permanent sticking of the belt, change of elastic properties of the belt, problems with estimating slip and sticking zones over the circumference of the pulley. Especially sensitive are more complex friction models such as: elastic/perfectly-plastic friction low (EPP) [7], Coulomb-like tri-linear creep-rate-dependent friction model [8, 10] or Dahl model [4].

2. The model

Fig. 1 shows the cross-section view of an exemplary poly-V belt 5pk, which is very often used in the automotive industry. In the case of one-sided belts, i.e. as shown in Figure 1, the pulleys are wrapped only with the working part of the belt (grooved side), but two types of tensioning rollers can be found contacting on each side of the belt – working or opposite, i.e. flat. Determining the position of the belt, e.g. for the purpose of discretising it, may be quite problematic.

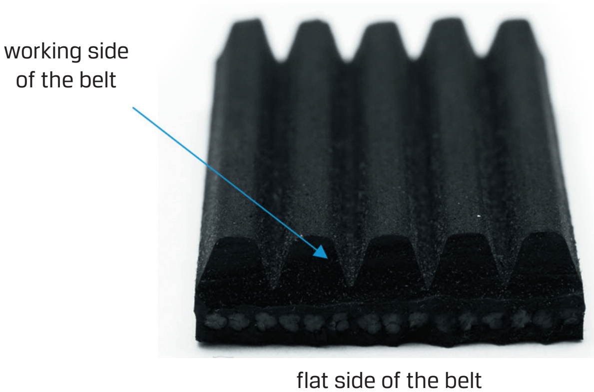

Fig. 1. Cross section of the belt

It was assumed that the wheels would be defined in order of occurrence in the transmission, in the clockwise direction, as shown in Fig. 2. Assuming that individual parts of the belt would also be considered in that order, it was also assumed that the wheel would be wrapped around the belt by its working side also in the clockwise direction. The example shows the wheels numbered as 1, 2, 3 and 5 working with the belt from its working side, while roller 4 is working with the belt from its flat side.

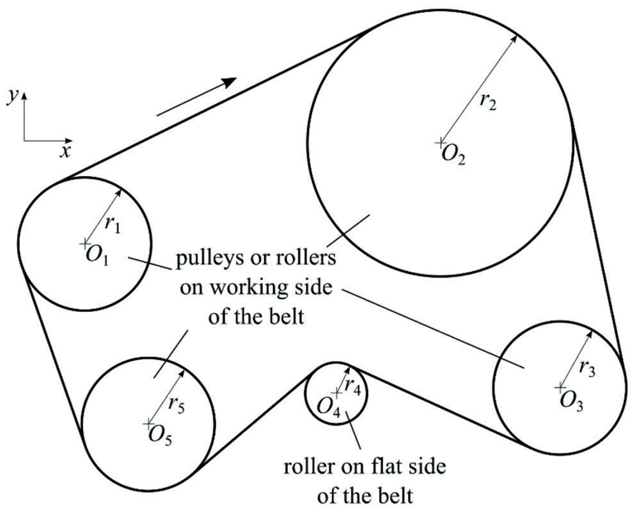

Fig. 2. The assumed order of pulleys and tensioning rollers shown based on an example of a four-pulley transmission with one tensioning roller

Fig. 3 shows four possible cases of belt wrapping of two adjacent wheels: the case of two belt pulleys interacting with the working side of the belt (Fig. 3a), two possible cases of the belt interacting with the pulley or tensioning roller from both the working and flat side (Fig. 3b: working part, then flat; Fig. 3c: flat part, then working part) and the case of rollers wrapped by the flat side of the belt (Fig. 3d). In each of these cases there are different angles of belt tension and length of belt part between the wheels.

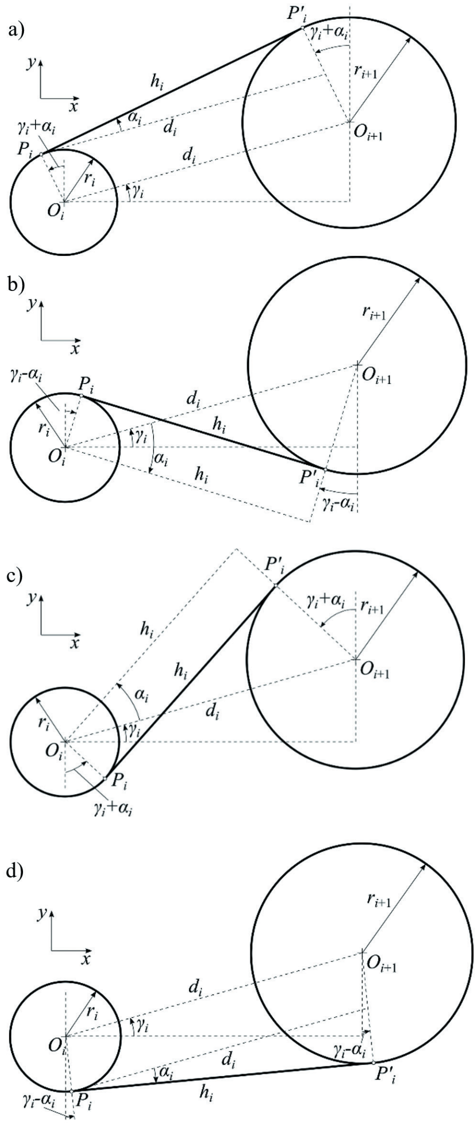

Fig. 3. Possible cases of two neighbouring pulleys with the belt

The figures show two neighbouring pulleys: with the centre in point O_i and radius r_i and with the centre in point O_{i+1} and radius r_{i+1}. The distance between the centres of the pulleys is d_i. It was additionally assumed that the part of the belt between the pulleys $\{i\}$ and $\{i+1\}$ would be presented by segment $P_i P_i'$. The length of the segment is h_i. The angle γ_i is measured between the segment $O_i O_{i+1}$ and the horizontal direction, whereas angle α_i represents the slope of segment $P_i P_i'$ in relation to the segment $O_i O_{i+1}$.

In addition, a special coefficient δ_i was introduced which can take on the following values:

$$\delta_i = \begin{cases} 1 & \text{in case of wrapping the pulley } \{i\} \text{ by the working side of the belt,} \\ -1 & \text{in case of wrapping the pulley } \{i\} \text{ by the flat side of the belt.} \end{cases} \tag{1}$$

For the cases shown in Fig. 3, the values of δ_i and δ_{i+1} are as follows:

- the case from Fig. 3a): $\delta_i = 1$, $\delta_{i+1} = 1$,
- the case from Fig. 3b): $\delta_i = 1$, $\delta_{i+1} = -1$,
- the case from Fig. 3c): $\delta_i = -1$, $\delta_{i+1} = 1$,
- the case from Fig. 3d): $\delta_i = -1$, $\delta_{i+1} = -1$.

In each case the distance between the centres of adjacent pulleys or rollers is:

$$d_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}\ , \tag{2}$$

where:

$$O_i = (x_i, y_i), \tag{3}$$

$$O_{i+1} = (x_{i+1}, y_{i+1}). \tag{4}$$

The length of the belt between the pulleys or rollers, assumed as the distance between points P_i and P_i', can be calculated from the following formula:

$$h_i = \sqrt{d_i^2 - d_{ri}^2}\ , \tag{5}$$

where:

$$d_{ri} = r_{i+1} - \delta_i \delta_{i+1} r_i. \tag{6}$$

The angle of inclination of the section determined by points P_i and P_i' is $\gamma_i + \delta_{i+1}\alpha_i$, and it can be calculated from the following trigonometric dependencies:

$$\sin\gamma_i = \frac{y_{i+1} - y_i}{d_i}\ , \tag{7a}$$

$$\sin\alpha_i = \frac{d_{ri}}{d_i}\ , \tag{7c}$$

$$\cos\gamma_i = \frac{x_{i+1} - x_i}{d_i}\ , \tag{7b}$$

$$\cos\alpha_i = \frac{h_i}{d_i}\ , \tag{7d}$$

$$\sin(\gamma_i + \delta_{i+1}\alpha_i) = \sin\gamma_i \cdot \cos\alpha_i + \delta_{i+1} \cdot \cos\gamma_i \cdot \sin\alpha_i\ , \tag{7e}$$

$$\cos(\gamma_i + \delta_{i+1}\alpha_i) = \cos\gamma_i \cdot \cos\alpha_i + \delta_{i+1} \cdot \sin\gamma_i \cdot \sin\alpha_i\ . \tag{7f}$$

The coordinates of points $P_i = \left(x_i^P, y_i^P\right)$ and $P_i' = \left(x_i^{P'}, y_i^{P'}\right)$ are therefore equal:

$$\begin{cases} x_i^P = x_i - \delta_i r_i \sin(\gamma_i + \delta_{i+1}\alpha_i) \\ y_i^P = y_i + \delta_i r_i \cos(\gamma_i + \delta_{i+1}\alpha_i) \end{cases} \tag{8a}$$

$$\begin{cases} x_i^{P'} = x_{i+1} - \delta_{i+1} r_{i+1} \sin(\gamma_i + \delta_{i+1}\alpha_i) \\ y_i^{P'} = y_{i+1} + \delta_{i+1} r_{i+1} \cos(\gamma_i + \delta_{i+1}\alpha_i) \end{cases} \tag{8b}$$

It was assumed that the belt would be divided into n_p points:

$$D_j = \left(x_j^D, y_j^D\right), \tag{9}$$

where:

$j = 1..n_p$ – number of the point.

It was also assumed that the points would be equidistant from one another (counting pulleys and rollers as arcs). The distance between points is therefore constant and can be described by the following formula:

$$h = \frac{l}{n_p}, \tag{10}$$

where:

l – total length of the belt.

The discretisation process of the belt is alternately divided into the following steps of analysis: of the part of the belt that surrounds the next pulley or roller and the loose part between them. It was assumed that the first point would be located at the beginning of the loose part of the belt between the first and second wheels. The discretisation process will thus end with discretising the points lying on the arc on the first wheel.

Fig. 4 shows the way of discretisation of the loose portion of the belt between neighbouring wheels or rollers.

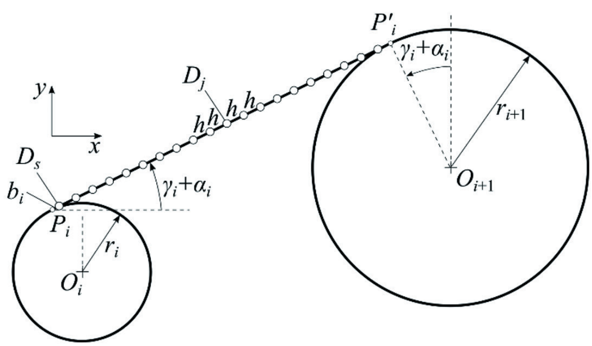

Fig. 4. Discretisation of the loose part of the belt between the wheels or rollers (case of contact of the working side of the belt on both wheels)

In the presented algorithm it is necessary to identify the first discretisation point on the analysed part of the belt. In Fig.4 this point is denoted as D_s. The next consecutive points, D_j, lie on the line defined by points P_i and P_i'. Their coordinates can be determined from the following relationships:

$$\begin{cases} x_j^D = x_P + \dfrac{a_j}{h_i}\left(x_{P'} - x_P\right) \\ y_j^D = y_P + \dfrac{a_j}{h_i}\left(y_{P'} - y_P\right) \end{cases} \tag{11}$$

where:

$$a_j = (j - s)h + b_i, \tag{12}$$

b_i – part of the length h between points $D_{s\text{-}1}$ and D_s, measured from point P_i to point D_s.

The discretisation process of the wrapping part of the belt is shown in Fig. 5.

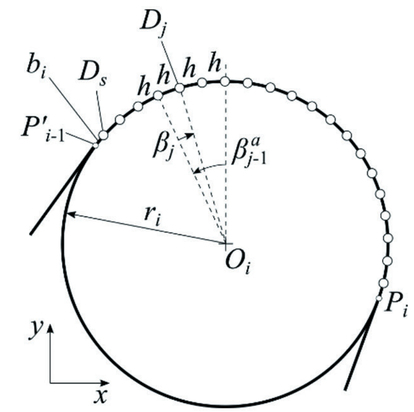

Fig. 5. Discretisation of the part of the belt that surrounds the pulley or roller (the case of the clockwise direction with the working side of the belt)

In this case, the position of the following D_j points is determined by the following coordinates:

$$\begin{cases} x_j^D = x_i - r_i \sin\left(\beta_{j-1}^a - \beta_j\right) \\ y_j^D = y_i + r_i \cos\left(\beta_{j-1}^a - \beta_j\right) \end{cases} \tag{13}$$

where:

$$\beta_j = \frac{\delta_i h}{r_i}, \tag{14}$$

$$\beta^{a}_{j-1} = \sum_{l=s}^{j-1} \beta_l + \frac{\delta_i b_i}{r_i} + \beta'_{i-1} \text{ - angle between } y \text{ axis and segment } O_i D_{j-1}, \tag{15}$$

$$\beta'_{i-1} = \gamma_{i-1} + \delta_i \alpha_{i-1}, \tag{16}$$

b_i – a portion of length h between points D_{s-1} and D_s already lying on the arc.

It should be emphasised that the thickness of the belt in the presented procedure is not taken into account. It was recognised that these values can easily be taken into account in the radius of the pulleys and rollers. However, it should be noted that this distance may be different if the wheel cooperates with the belt from its working side and is different from its flat side.

3. Sample results of an automobile transmission

A computer program calculating the coordinates of points D_j was developed based on the formulas presented here. The program's interface is shown in Fig. 6.

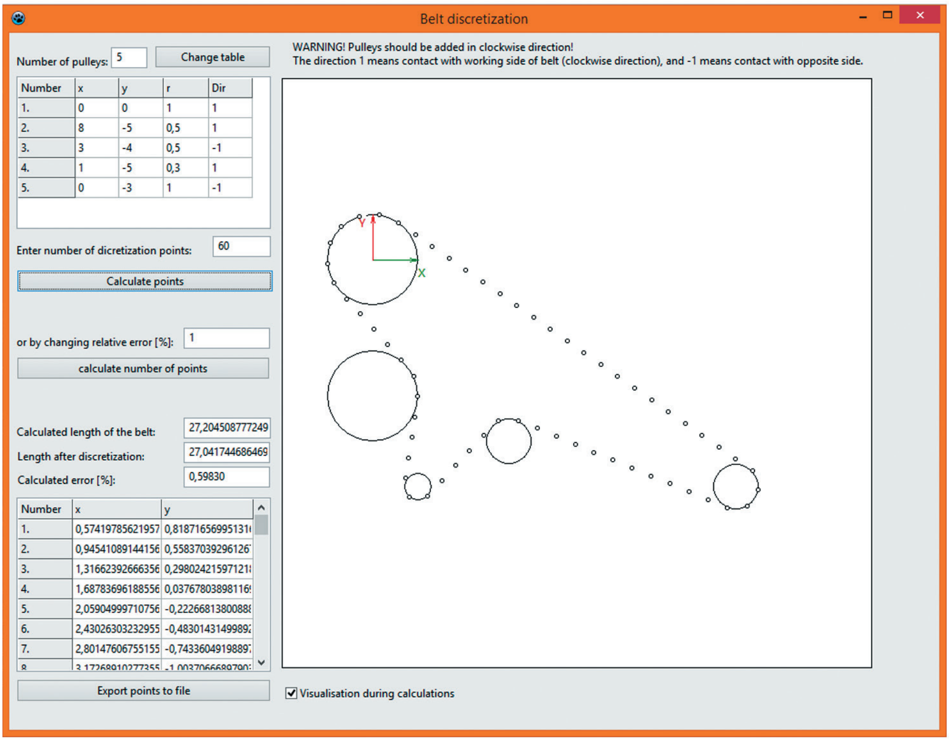

Fig. 6. Computer program developed for belt discretisation

The program allows to enter any number of pulleys or rollers, with the choice of which side of the belt will work with the pulley or roller.

In addition, the program includes calculating the length of the belt, taking into account the appropriate lengths of arcs on the wheels and rolls (its theoretical length before discretisation but also the length after its discretisation). As a difference of these lengths, a measure of the accuracy of the discretisation process was taken into account and expressed as a relative error as:

$$e(n_p) = \frac{l - l_D(n_p)}{l}, \tag{17}$$

where:

l_D – length of the belt after discretisation.

The calculated points can be exported into a file and used in belt dynamics models with discrete belts.

Fig. 7 shows an exemplary analysis of the transmission rotating alternator pulley and the coolant pump from the engine shaft in a diesel automobile.

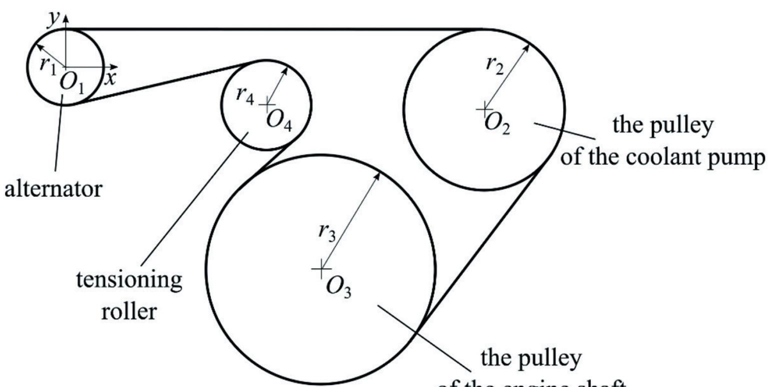

Fig. 7. Sample automobile transmission

Fig. 8 presents the relationship between the resulting relative error and the number of points. There is a strong nonlinear relationship, e.g. it was calculated that for this configuration of the transmission only 38 points were needed to achieve a discretisation accuracy of 1%, while a tenfold reduction of 0.1% required only 120 discretion points.

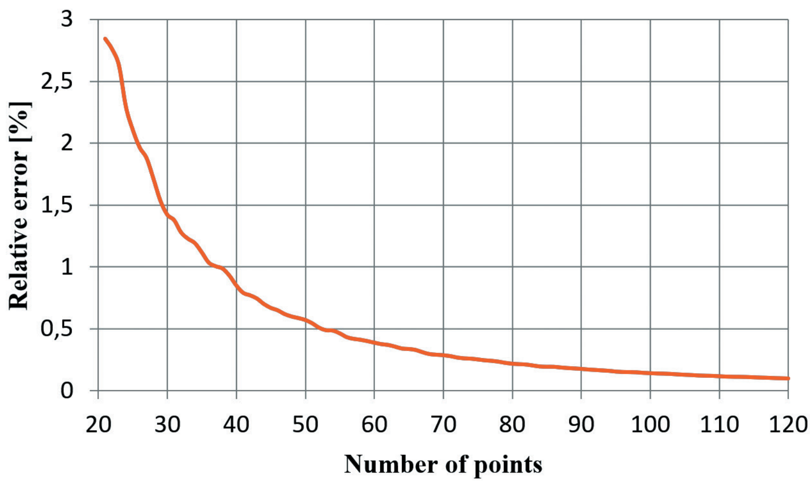

Fig. 8. Dependence of the relative length error after discretisation of the belt expressed as a function of the number of assumed points

Fig. 9 presents the positions of discretisation points for a calculated number of 38 and 120 points.

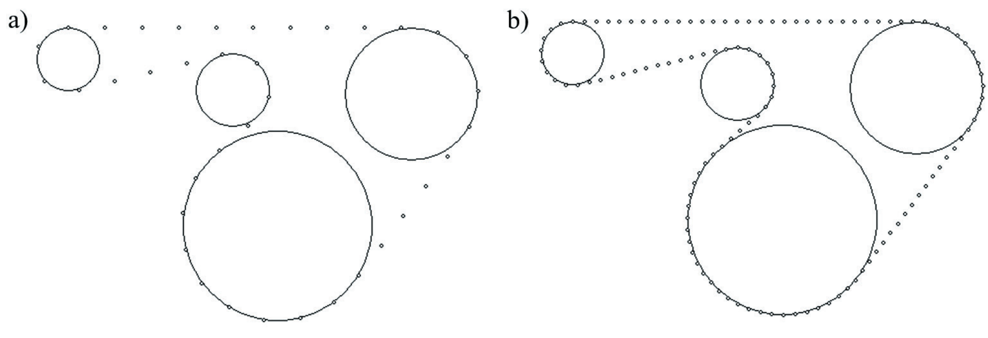

Fig. 9. Positions of the calculated points in the case of:
a) n_p = 38, b) n_p = 120

Fig. 9a clearly shows that despite the fact that a small error of 1% has been accepted, the number of 38 points may be too small. The number of discretisation points is only 3 on the arcs surrounding the alternator wheel and the roll. This leads to a considerable shortening of the distance between them, relative to length h measured on the circumferences.

It was therefore decided to assume a different criterion for the selection of the number of points as: for every $j = 1...n_{p-1}$:

$$e(n_p) = \frac{h - \min\left|D_j\ D_{j+1}\right|}{h} = \frac{h - \min\left(\sqrt{\left(x_{j+1}^D - x_j^D\right)^2 + \left(y_{j+1}^D - y_j^D\right)^2}\right)}{h}. \quad (18)$$

As can be observed from the above relationship, it was decided to accept the smallest distance between neighbouring points as the criterion of the selection of points.

As expected, after recalculation the number of points increased for the new criterion that had been assumed. The following results were obtained: the error of the minimum length of the belt was no greater than 1% for the 90 points assumed and was not to exceed 0.1% in the assumed 284 points. The results of the belt discretisation process, for the revised criterion of the number of points, are presented in Fig. 10.

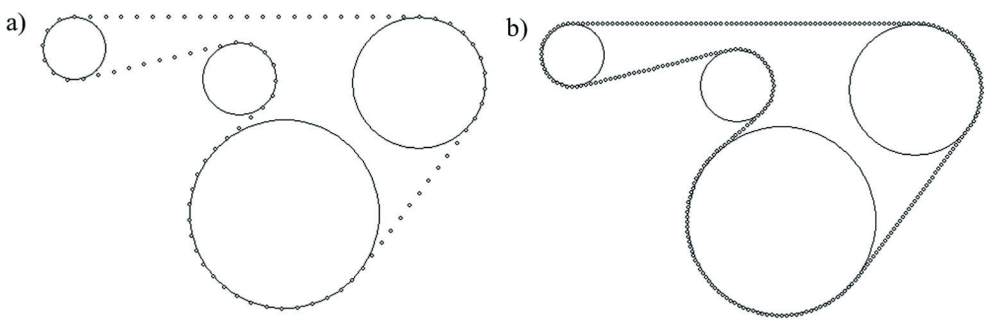

Fig. 10. Positions of the calculated points in the case of:
a) n_p = 90, b) n_p = 284

Fig. 11 shows a graph of the error comparison for dependencies (17) and (18). As can be observed, the newly adopted criterion can be considered more "restrictive" because the error is greater.

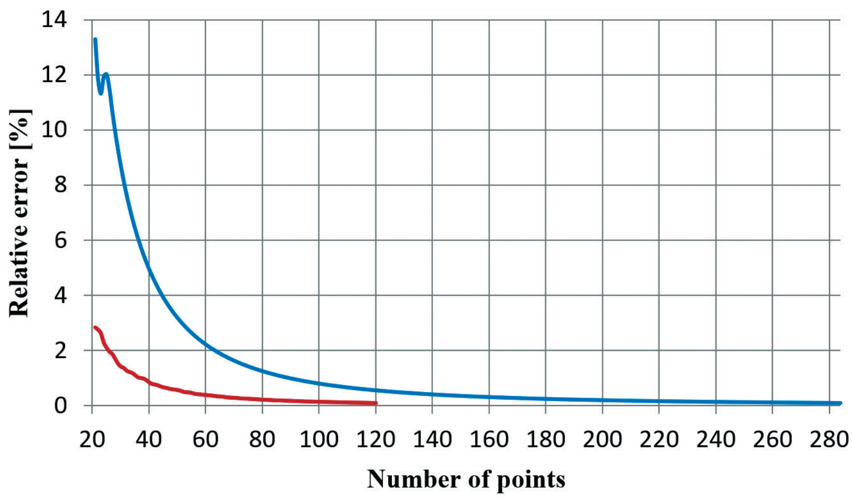

Fig. 11. Dependence of the relative length error after discretisation of the belt expressed as a function of the number of assumed points – a comparison of two assumed criteria: from dependence (17), from dependence (18)

Also note the disturbance of the newly acquired characteristic at point n_p = 24. Assuming that the first discretion point always lies in the same place, the points may also be so arranged that the calculated error will be greater after increasing the number of points. As can be guessed, this is the case for the smallest wheel, i.e. the alternator wheel.

4. Conclusions

As can be observed, the problem of discretisation of the belt, particularly in cases of a multi-pulley transmission, may be troublesome. It is particularly important to specify the appropriate evaluation criterion for how many points will discretise the belt. By adopting the appropriate criterion (e.g. as suggested in this paper), we have to bear in mind that the number of points is determined by the specifics of the transmission itself, i.e. not only its size but also the size of the smallest pulleys and tension rollers, or the number of consecutive changes in their wrap direction. It is important to remember that too small a number of elements may result in poorly reproduced arcs on wheels and small radius rolls and too large a disproportion between the distances between the points. As has been shown in this paper, the assumption of too large an error can also cause disturbances in the process of selecting the number of points for discretisation. The objective function may not be monotonous in the first phase of this process.

The methodology presented here does not take into account that the modelled belt in the transmission should be pre-tensioned. If the dynamic model includes longitudinal spring-damping elements, this can be achieved simply by stretching and shortening the beams between them. Another way is to reduce the rays of the pulleys and rollers during discretisation and to re-enlarge them in the model. In this way the beams will pre-penetrate the bodies of the pulleys and the rollers, and tighten the belt. This second method may also be useful for belt models in the form of beams connected by rotational joints (in this case there is no spring-damping element). In both cases, however, it should be borne in mind that prior to a fundamental analysis of dynamics, the system should be pre-stabilised, preferably by finding its static equilibrium.

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