MACROSCOPIC MODEL OF ENERGY DEMAND TO OVERCOME ROAD GRADE RESISTANCE OF VEHICLE MOTION

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Summary

The article discusses the problems of modeling of energy requirements to overcome grade resistance in vehicle motion in macroscopic scale simulations. Authors describe ways of modeling this requirement with the use of forward and backward facing models in microscopic simulations. The data limitation problem is defined, i.e. data only available in sparse values, describing only altitude of nodes between sections at length of several dozen to several hundred meters. Authors propose to use potential energy change as basic approximation of energy demand to overcome grade resistance and show estimated impact of motor and inverter efficiency on electric energy consumption. More accurate model is proposed based on additional road section parameter – hypothetical road peak. Based on carried out verification with an example of route and speed profile, significant improvement of energy estimation made with the three-parameter section altitude description is shown.

Keywords: macroscopic traffic models, macroscopic models of energy consumption, road grade resistance forces

1. Object under test

The amount of energy consumed by vehicle to cover a given road section depends on requirements for mechanical energy on wheels and efficiency of the vehicle drivetrain. Requirements for mechanical energy on wheels of the vehicle are determined by variable in time requirement for motive power necessary to overcome motion resistance forces and by time which these forces act for. With constant level of those requirements it is the product of power and time for which energy requirements were defined. The value of resistance forces can be estimated based on relatively simple formulas presented in many books describing fundamentals of vehicle dynamics and especially its longitudinal dynamics [5,7,13,14,19]. But this is simple methodology only when assuming constant conditions of vehicle motion. In real life parameters of those equations are subject to constant change and determining them is actual problem. The ones varying the most are vehicle speed, road gradient, wind speed and direction. Other parameters can also be subject to change, such as aerodynamic drag coefficient.

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Because of that, in order to estimate realistic energy requirement, it is necessary to know variability of:

- speed described by speed profile, which has impact on aerodynamic drag, and intensity of its change impacts inertia resistance.
- road gradient, defining grade resistance.

The same speed and road profile driven by two identical cars should result in the same energy consumption. If efficiency of energy transfer via drivetrain is different, then consumption of energy delivered to the drive train will be different. But in all cases amount of energy delivered to the drive train is higher than amount of mechanical energy required to overcome resistances of motion. However it can be accepted that mechanical energy requirement is fundamental factor contributing energy consumption and sets its minimal value.

Real life speed profiles performed by vehicles in traffic have random nature. This is a result of random character of factors influencing speed achieved by vehicle. Such factors include random character of external restrictions (for example speed of leading vehicle, incidents in traffic, traffic lights) and internal limits (driving style impacting desired speed and rate of acceleration, dynamic characteristics of vehicle). Among many possible speed profiles some groups with similar characteristics can be found, for example based on the mean speed, energy consumption, length, scope of acceleration values, etc. Based on those similarities it is possible to propose speed profiles that are representatives of given conditions/area of a vehicle usage. This approach is utilized to assess fuel/energy consumption by vehicles that are tested with selected speed cycles in laboratory conditions [2].

2. Modeling of energy consumption

It is possible to perform simulation study of energy consumption using models of a drivetrain and resistance forces of motion. For proper modeling of resistance forces it is substantial to define both speed profile and road gradient profile – different resolutions of information are used. For example 0.1 s or 1 s for speed profile is used and for road profile every 1 m, 10, 50, 100 m or distance travelled in a time of 1 s is used.

For models of drive train the one important aspect is structure of this system and characteristics of individual components and algorithms controlling each part and the whole system. This data is specific for each type of a drivetrain, its structure, control systems and even manual control made by driver. This is the reason why in this case universal models are harder to obtain – it is possible to make one based on some sort of simplification, for example concerning structure, but often specific solutions require knowledge of detailed parameters of given technical solution (gear ratio, amount of internal losses, characteristics of engine or motor and control algorithms). Often due to technical complexity of technical solutions some of these parameters or characteristics have to be determined with experimental methods.

Additional factor that influences energy consumption of vehicle exploitation (not only vehicle motion) is energy consumption by on-board equipment, not directly connected with movement of vehicle, but used during travel and short stops. Most important of those factors are additional receivers of electric or mechanical energy – power steering, air conditioning, lights. In case of electric cars substantial energy consumption comes from heating, which in cars powered by internal combustion engine comes from engine waste heat, in absence of that engine this energy has to be supplied from main battery pack [1].

In a literature and an existing software there are two methods of estimating energy consumption with simulation [10, 11] – Fig. 1:

1. forward-facing – models in which input consists of steering signal of motive power and road gradient profile, energy consumption and achieved vehicle dynamics are dependent on parameters of drivetrain and whole vehicle. To estimate energy consumption for set speed profile it is important to include model of driver behavior as controller executing given speed profile,
2. backward-facing – models in which the input consists of road profile and executed speed profile. Based on this information data about engine rotational speed and torque are estimated along drivetrain up to engine or motor, in this approach there is no need for driver model.

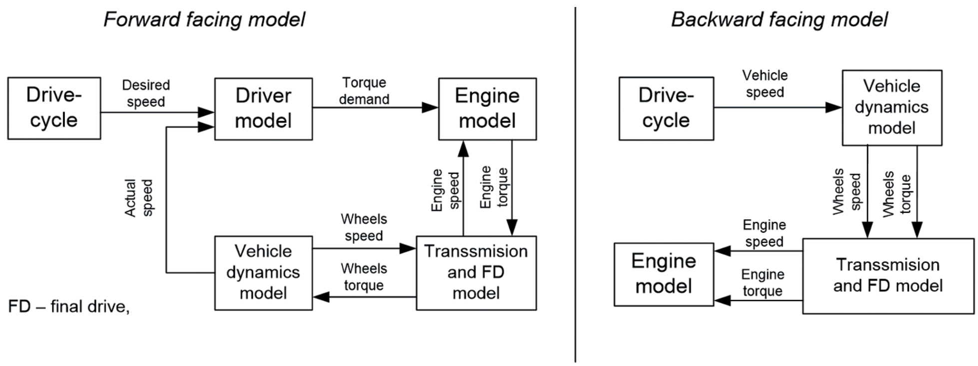


Fig. 1. Forward and backward facing models (based on [11])

Both approaches are used in simulations, with time step defined by resolution of speed profile – predominantly 1 s, but others are also used, for example 0.1 s. Those type of simulations are used to estimate energy consumption by single vehicle, for example during design of control system responsible for energy management with tests of real vehicle used as reference point or verification.

Another approach to energy consumption modeling is required for simulation of road traffic, that does not focus on analyzing single vehicle but instead deals with many vehicles at once. Models used in traffic simulation can be divided into three groups: microscopic, mesoscopic and macroscopic [9]. Microscopic models allow simulation with highest accuracy – in relation to single vehicle, macroscopic models are used for simulation of whole fragments of road network.

Similar level of accuracy is required from models of energy consumption. Papers [3, 18] presents classification of road traffic models and models of fuel consumption along with models of vehicle emissions. Macroscopic models of fuel consumption are models based on speed profile parameters (speed profile models) or models based on the state of engine work and accurate speed profile (modal emission model) – this kind of model uses information about engines operating points (torque and rotation speed).

In macroscopic traffic models simulation step can be determined by for example road sections between points of certain events, like change of direction. This means that single analyzed section is distance between two intersections and time steps are reflecting the time vehicle spends on one road section. For sections like this knowledge about speed profile is limited to value of mean speed and information about road profile is limited to height difference between nodes of road network. Authors of [18] distinguish following macroscopic model types:

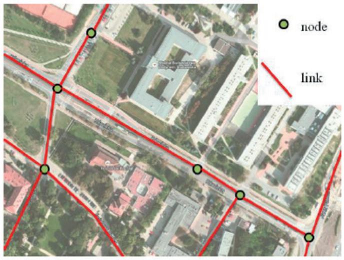
- area-wide models,
- average speed models,
- traffic situation models,
- traffic variable models.

3. Assumptions and goals for building macroscopic model for road gradient

Authors of this paper in earlier work developed macroscopic model of energy consumption based on vehicle's average speed. It shows relation between mean speed and energy consumption. This model was developed based on the study of speed profiles performed in real world traffic and calculations of resulted energy consumption performed by microscopic model based on operating points of electric motor to estimate efficiency end electric energy requirements. Exemplary division of route into sections and achieved values of mean speed are shown in figure 2.

Developed macroscopic model did not include diversity of road altitude or road gradient profile. During research based on simulation [12] it was found that due to adopted length of sections, of several dozen to several hundred meters, it is possible to obtain inaccuracy of estimated energy consumption of magnitude around 15% – Fig. 3. This error can decrease or increase estimated value depending on direction of travel, which results from positive or negative gradient of traveled road.

Character of input data to a macroscopic model compared to the data used in microscopic model is shown in figure 4. This data consists of values of mean speed on road section and height of each node.



section length	quantity
0 - 50 m	50
50 - 100 m	161
100 - 200 m	307
200 - 300 m	218
300 - 500 m	331
500 - 1000 m	114
over 1000 m	36

Fig. 2. Division of roads into sections and information about number of sections sorted by length [16]

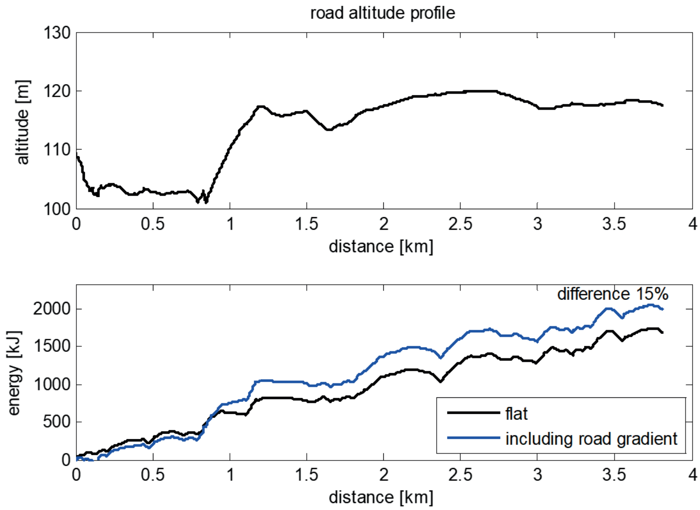


Fig. 3. Example of road profile and difference between energy consumption between estimation with and without grade resistance

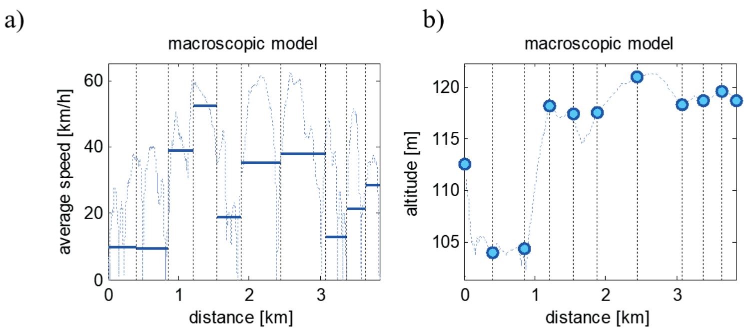


Fig. 4. Input data for macroscopic model – average speed per section and road altitude at nodes

In practical use average speed values can be calculated based on information about length of section and time how long vehicle remained on a given section. Graph in figure 4a beside average speed values presented in a relation to section length shows data recorded in continuous fashion with resolution of 1 s – recorded speed profile in a relation to section length. Figure 4 b shows information about road altitude at road network nodes, referenced to road profile based on data from GPS receiver recording speed profile and road altitude at every 1s.

Limitation of information concerning speed profile and road gradient profile leads to question how does this lack of accurate data influence researchers' ability to estimate energy consumption required to overcome motion resistance. Development of models for resistances of inertia, roll and aerodynamic drag was shown in previous paper [16].

In this paper the authors present study assessing the capability of macroscopic modeling of mechanical energy requirements required to overcome grade resistance. Particular explored topics are:

- analysis of specifics of energy requirement to overcome grade resistance for different road grade profiles,
- relation between final values of energy consumption to overcome grade resistance for whole road section resulting from resolution of macroscopic model,
- formulation of macroscopic model estimating energy requirements to overcome grade resistance for whole road section, from resolution of macroscopic model, based on knowledge of only selected road section parameters instead of detailed grade profile,
- comparison of estimation of energy requirements to overcome grade resistance by macroscopic model with reference value obtained from microscopic model.

4. Analysis of consideration of grade resistance in macroscopic models

Problem analyzed in this paper was studied by other researchers. Based on analysis of chosen works [4, 6, 17] it can be stated that in macroscopic simulations following approaches to include grade resistance in estimation of energy requirement to overcome this resistance are possible:

1. utilization of data about road grade with lower resolution than microscopic models – for example 100 m,
2. estimation of required energy based on change of potential energy of vehicle between begining and ending points of section,
3. introduction of additional conversion factor based on impact of grade resistance on increase of average motion resistance.

First approach was presented in paper [4], based on example of 50 km long highway route. Its author found that energy consumption estimation is lowered by 30% if grade resistance is not taken into account. In this paper author proposes method to incorporate grade resistance to energy calculation. This method is based on calculation of motive power required to overcome resistance forces, including grade resistance, based on attitude data with 100 m resolution. Negative road inclination (driving downhill and resulting energy recuperation) were ignored in order to increase safety margin of energy requirement estimation.

Second approach, for situation when detailed road profile is unavailable, was described in [17]. Estimation of energy required to overcome grade resistance is based on the change of potential energy between characteristic points of route. In described method required energy is calculated based on number of factors (average speed, number of stops, usage of onboard equipment, etc.), including change of potential energy of vehicle.

Third approach is described in [6] as method to estimate emission of harmful exhaust components by fleet of vehicles. This method is based on calculation of emissions from vehicle driving on flat terrain, and subsequently multiplying obtained result by appropriate coefficient. Values of this coefficient depends on road grade category and average speed of a vehicle.

5. Study of specifics of energy requirements to overcome grade resistance for selected configurations of grade profiles

Simulation study was performed to asses changes of energy requirement to overcome grade resistance analyzing the changes in drives over 1 km long test section based on microscopic model and assessing possibilities of estimation of this energy requirement based on change of potential energy. Influence of configuration of grade profile on electric energy consumption by electric traction motor was also analyzed. This analysis was performed based on information about efficiency of electric motor operating as a motor and as a generator. Impact of battery efficiency and other onboard equipment was omitted, thus obtained data cannot serve as direct estimation of energy consumption by electric vehicle.

For this study 5 variants of test section were defined, each consisting of 4 fragments, two always flat sections A and D with length of 10% of whole test section as well as two sections of varying grade, each with length of 40% of whole section – figure 5.

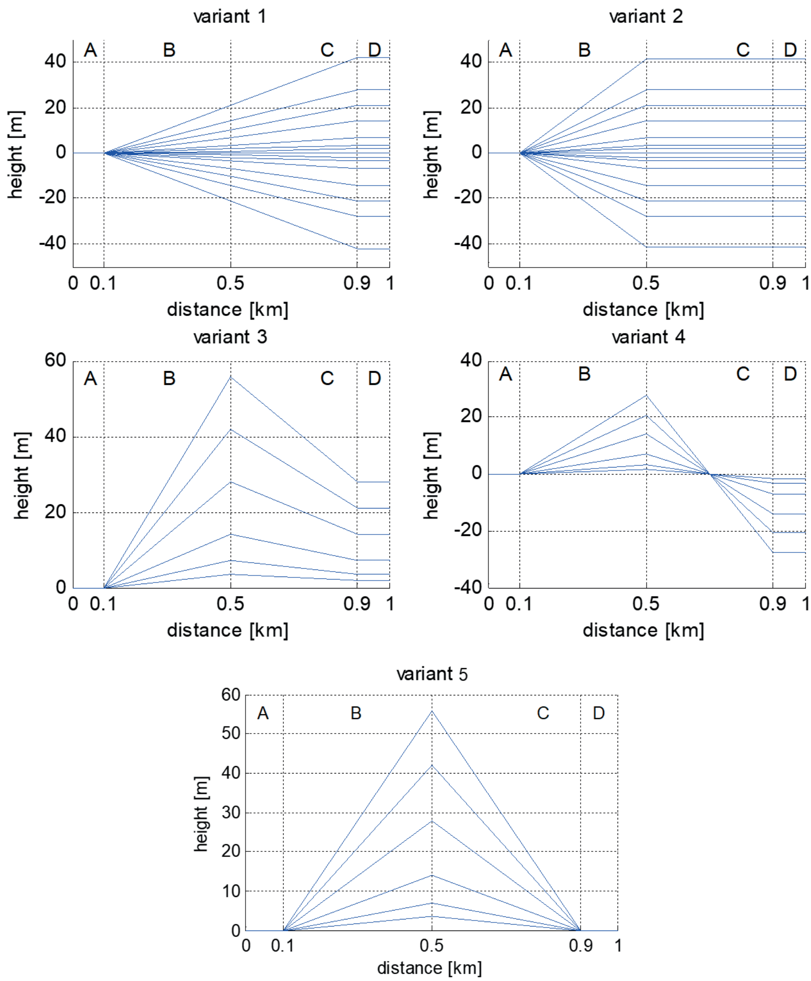


Fig. 5. Road profiles used in simulation study variants 1-5.

Consecutive variants of sections B and C were chosen in a way to analyze impact of single change of road altitude between beginning and end of test section, and an impact of road grade that is not direct result of height difference:

1. variant 1 – change of altitude takes place on fragments B and C,
2. variant 2 – road inclination is present only on fragment B, resulting in more steep road grade to achieve same height difference, fragment C is flat,
3. variant 3 – fragments B and C have different inclination, resulting in ending point on greater height than beginning,

4. variant 4 – fragments B and C have different inclination, resulting in ending point on lower height than beginning,
5. variant 5 – fragments B and C have inclination of same value but opposing sign, resulting in ending point on same height as beginning.

For variants 1 – 4 angular values of road fragments are chosen to obtain test sections of same average grade, but different road profile. Average grade is defined as ratio between height difference of beginning and ending points to length of test section, measured in percent.

In an experimental study requirement for mechanical energy required to overcome grade drag was studied, which is the reason in first place other forces of resistance were omitted. To further simplify interpretation of the results of, the simulation was to be limited to constant speed, with two speed values chosen – 40 and 60 km/h. During simulation electric energy delivered to vehicle traction motor was also estimated based on the efficiency characteristics of motor and inverter paired with other parameters of Nissan Leaf battery electric vehicle - Fig. 6 [3,8,15].

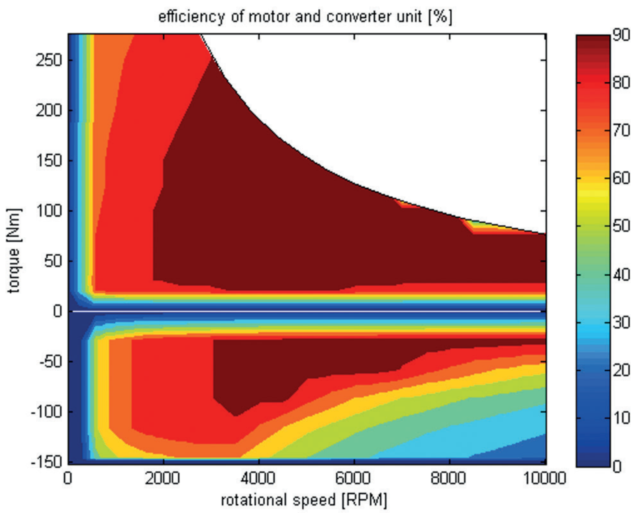


Fig. 6. Efficiency of motor and converter used in microscopic model

During the **first simulation experiment** three road profiles were analyzed – all with same value of average road grade of 1.397%, but different road profiles:

– test 1 – inclination of fragments B and C was 1°,
– test 2 – inclination of fragment B was 2°, with flat fragment C,
– test 3 – inclination of fragment B was 4°, inclination of fragment C was -2°.

Graph in figure 7 shows comparison of simulation results for this experiment. Similarity between shape of charts describing change of altitude and change of mechanical energy requirements is result of connection between the change of potential energy of vehicle and the change of its altitude. Furthermore the change of potential energy is dependent only on altitude change and not on path of that change. This means that mechanical energy required to overcome grade resistance is equal to change of potential energy of vehicle. This is reason why final value of energy required to overcome grade resistance in all three analyzed variants is the same.

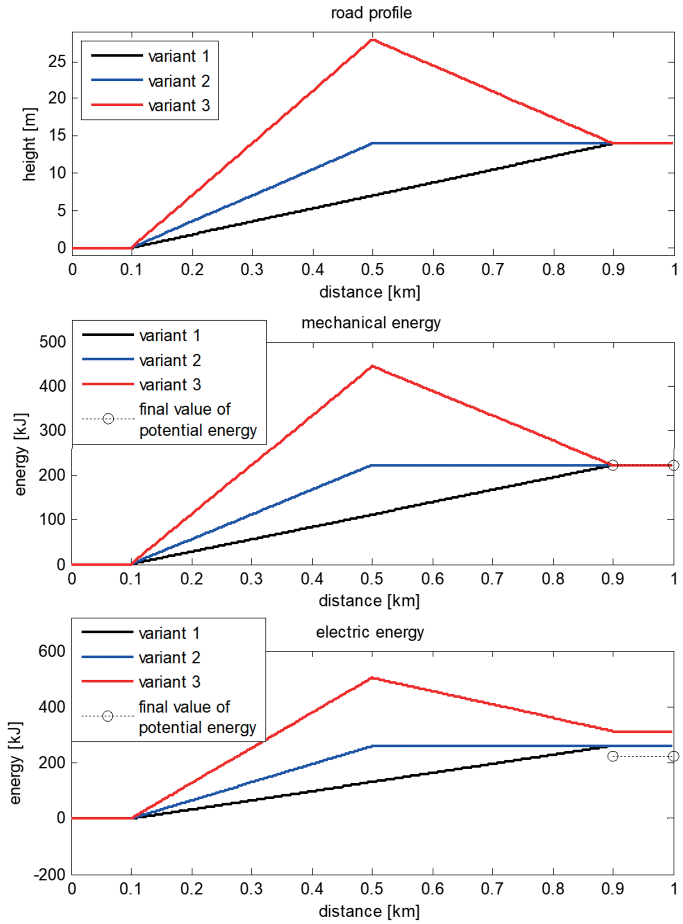


Fig. 7. Comparison of mechanical energy requirement and electrical energy supplied to motor to overcome grade resistance for road profiles of same average grade

Values of required electric energy display similar behavior when road inclination has constant sign along the whole route (positive or negative, variants 1 and 2), in this case energy requirement is result of change in vehicles potential energy and due to the similarity of values of motor efficiency, the road profile has small impact on this value.

If on considered road section fragments with both positive and negative inclination are present instead, then it becomes important to take into account energy conversion in two directions: electric energy to mechanical (driving uphill) and mechanical energy to electric (driving downhill and energy recuperation). This causes overall increase in energy consumption or decreased recovery of energy. While going uphill energy consumed from energy reservoir (battery) is greater than mechanical energy delivered to wheels, while going downhill electric energy fed to battery is smaller than mechanical energy recovered from wheels.

During **second experiment** authors analyzed impact of road inclination on required mechanical and electrical energy during ride with constant speed of 40 km/h. This analysis was based on first variant of road profile with average inclination ranging from -4.2% to 4.2%. Results are presented in figure 8 which illustrates relationship between consumed energy and average road grade. Energy requirement including other resistance forces has similar shape, just raised by value of energy required to overcome these forces.

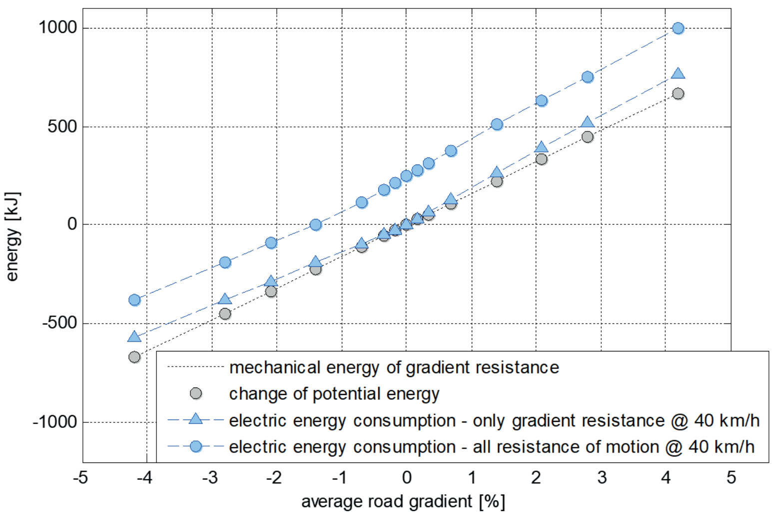


Fig. 8. Electric energy consumption from different road grade on test section

Third experiment was performed based on fifth variant of road profile, i.e. beginning and end of test section is on same altitude. In this case mechanical energy requirement to overcome grade resistance is independent on inclination values and always is zero for the whole section. However results of electric energy requirements indicate dependence on road profile and its value increases with increased road inclination. This is the result of different losses in energy conversion as mentioned earlier. This experiment clearly shows that information limited to change of height between extreme points of travel, or road sections is inadequate for accurate estimation of energy consumption for grade resistance – Fig. 9.

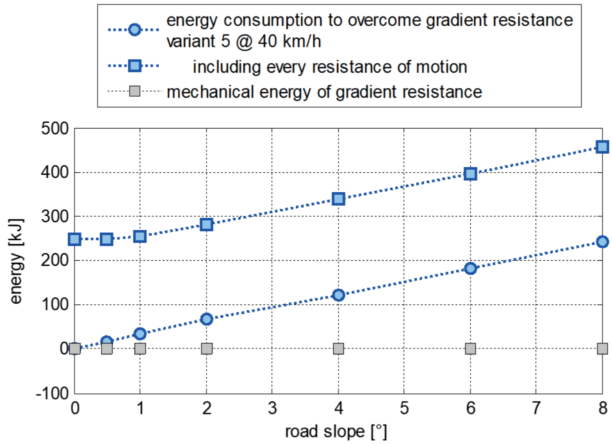


Fig. 9. Impact of road inclination on test section with average inclination equal zero on electric energy consumption

6. Macroscopic model of grade resistance impact on vehicle energy demand

As a result of an analysis described in previous part of the article, authors proposed the model of grade resistance impact on energy consumption of vehicle designed to work in macroscopic simulations. This model is based on assumption that energy E_i required to overcome grade resistance on road section between two nodes (i and i+1) is sum of energy E_{Vi} required to overcome resistance forces arising from speed profile (aerodynamic drag, inertia) along with roll resistance (model described in [11]) and energy E_{hi} required to overcome grade resistance (arising from road profile):

$$E_i = E_{Vi} + E_{hi} \tag{1}$$

Proposed model of required mechanical energy to overcome grade resistance assumes equivalence between this energy and change of potential energy resulting from altitude change between beginning point of section h_i and ending point h_{i+1}.

$$E_{h\,i} = mg(h_{i+1} - h_i) \tag{2}$$

In the simplest version input information consist of altitude change between ends of each section defined by resolution of macroscopic model. In some way it is a low resolution representation of a road profile – described by characteristic points a dozen to several hundred meters apart of each other.

In the case of estimating requirement for electric energy the model must be supplemented by inclusion of varying drivetrain efficiency. Depending on the direction of the energy flow different operation points of efficiency characteristics are used. Based on information about altitude of begining point h_i and ending point h_{i+1} coupled with efficiency of energy conversion from electric to mechanical $\eta_{E \to M}$(motor driving the vehicle) and from mechanical to electrical $\eta_{M \to E}$ (motor acting as generator and slowing vehicle down), electric energy consumption can be estimated based on formula:

$$E_{h\,i} = \begin{cases} mg(h_{i+1} - h_i)\eta_{M\to E} & if\ h_{i+1} < h_i \\ mg(h_{i+1} - h_i)\frac{1}{\eta_{E\to M}} & if\ h_{i+1} > h_i \end{cases} \tag{3}$$

In this approach whole section is treated as a indivisible element, thus efficiency values are averaged for the whole section. It is important to note that the result of calculations is electric energy. In calculations for uphill driving ($h_{i+1} > h_i$) inverse of efficiency value $\eta_{E \to M}$ is used, because its value is determined for conversion of electrical energy to mechanical.

In order to properly estimate average efficiency it is important to know the contribution of positive and negative inclination fragments on road section. In order to find proper average the authors propose to use the height of aggregated uphill driving h_+ and height of aggregated downhill driving $h\_$ on road section. This approach is based on treating a section as if it would consist of two parts, one with only uphill driving and second with only downhill driving (figure 10). This way energy flow in both directions can be taken into account without increasing resolution of calculations. Energy to overcome grade resistance can be calculated with the following formula:

$$E_{hi} = mgh_+ \frac{1}{\eta_{E\to M}} + mgh\_\eta_{M\to E} \tag{4}$$

To further limit the amount of additional information required to describe road profile authors propose to use a single additional parameter to information about altitude of the beginning and end of the section – hypothetical peak height h'. This parameter describes height relative to higher section end that would be reached by a vehicle if downhill driving would be eliminated from section (figure 10).

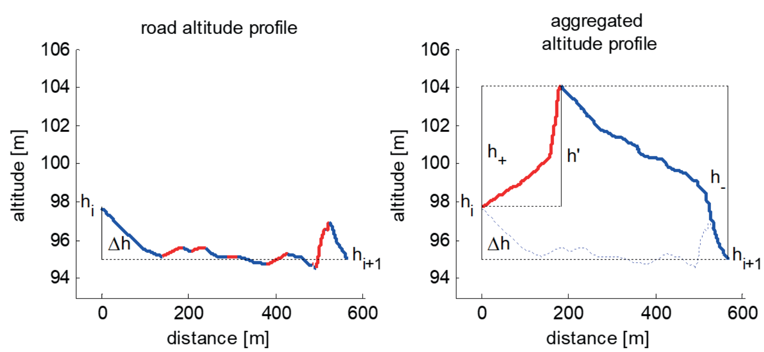


Figure 10. Conception of calculation of hypothetical peak height h'

The advantage of this approach is a limitation of additional parameters required to describe each road section to only one value. At the same time calculation of uphill and downhill drive heights is possible in both directions of travel. Downside of this approach lies in greater work required and bigger amount of data in preparation of input information – computational costs are decreased only during simulation and calculations of energy consumption.

To calculate values of aggregated height of uphill and downhill driving following formulas can be used:

$$for\,\Delta h > 0\ \begin{cases} h_{+} = \Delta h + h' \\ h_{-} = h' \end{cases} \tag{5}$$

$$for\,\Delta h < 0\ \begin{cases} h_{+} = h' \\ h_{-} = \Delta h + h' \end{cases} \tag{6}$$

$$for\,\Delta h = 0\ \begin{cases} h_{+} = h' \\ h_{-} = h' \end{cases} \tag{7}$$

where:

Δh – height difference between beginning and end of section,

h' – hypothetical peak height,

h_{+} – height of aggregated uphill driving,

h_{-} – height of aggregated downhill driving.

6.1. Verification of macroscopic model

In order to assess the quality of estimation of requirement for mechanical energy and electric energy consumption by traction motor a simulation test was performed, in which results of different models were compared. Results of microscopic simulation were used as reference data for verification of three macroscopic models. Microscopic model had time resolution of 1 s, and macroscopic models resolution was based on length of road sections between intersections.

Macroscopic models used were as follows:

1. 2-point model – based on altitude information of each end of section,
2. 3-point model – based on information of altitude of each end and also of middle point of each section,
3. hypothetical peak height model – based on altitude information of each end of section supplemented with hypothetical peak height described in previous part of article.

Plots shown in figure 11 illustrate obtained results allowing comparison between one another.

For this study it was assumed that microscopic model of energy requirements is sufficiently accurate and its use as reference point is dictated by its main difference from macroscopic models, i.e. incorporation of detailed speed and road profiles instead of data limited to average values for route sections.

Analysis of obtained results show that direct utilization of potential energy change allow to estimate mechanical energy required to overcome grade resistance and estimate electric energy required by traction motor to overcome said resistance – analysis of efficiency of electric motor and inverter suggest that average efficiency is of 80-90% magnitude in case of rather unified road grade.

In case of more elaborate road profiles this accuracy will drop and to improve it is necessary to use additional information about road profile, such as altitude of section central point or hypothetical peak height. However such approach requires preparation of additional information about each road section. This means that more detailed information about road altitude is required during data preparation and based on this data road parameters must be calculated before commencement of simulation. Such preparation can be performed only once and obtained road data can be used during multiple simulations. Overall it allows to decrease computational costs of simulation during its run and keep number of required parameters describing road network limited, which is important for macroscopic simulations of whole fleets of vehicles

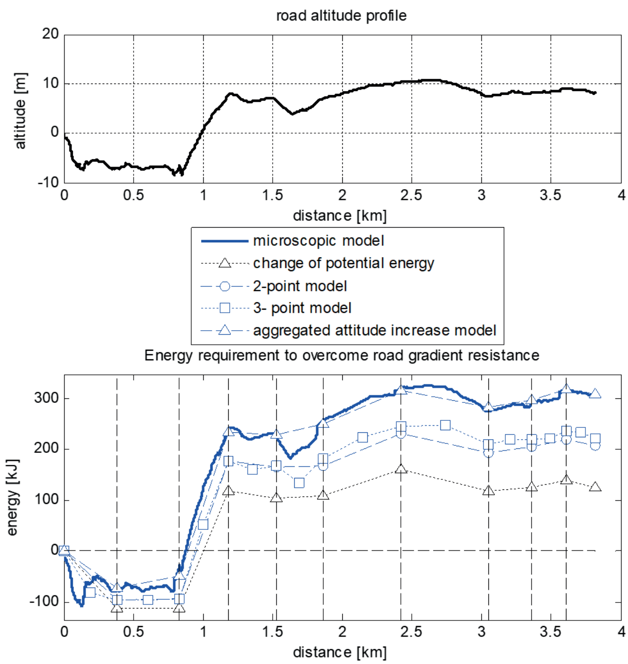


Fig. 11. Comparison of estimation of energy required to overcome grade resistance obtained from different models

7. Summary

Proposed macroscopic model of energy requirements to overcome grade resistance can be utilized to improve accuracy of macroscopic simulations of energy consumption by vehicles, and paired with a model of electric battery can improve results of macroscopic models of traffic used for forecasting available range, charging schedule or selection of route optimal in terms of energy consumption. Upon wide access to digital topographic data, creation of macroscopic model of road network containing information about altitude of characteristic points and hypothetical road peak is a matter of developing proper IT tool. This process is required only upon gathering the data, and can be re-used for multiple speed profiles.

It is worth noting, that even the use of only potential energy change to include impact of road grade can significantly increase accuracy of energy requirements estimation. It is important to note that purpose of such models is to combine simple and fast calculations with best accuracy of statistical estimation of energy consumption. Paired with input based on limited data these models cannot compete in accuracy terms with macroscopic models and comparison of energy consumption by specific vehicle with simulation results. Accuracy of these models is evaluated based on statistical analysis of large group of vehicles or long exploitation of handful of vehicles where cumulative impact of grade resistance can be assessed.

Purposefulness of this type of simulations can be exemplified by comparison of energy estimation from this article, where difference of energy consumption with and without including grade resistance amounted to 15%, for route of average around 0.5% (altitude change of 19.2 m) at distance of 3.8 km.

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